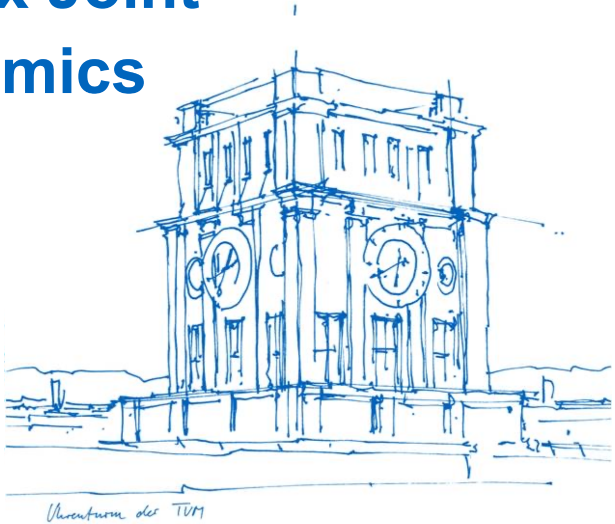
Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

Sparse Identification of Unknown Equation of Motion Terms Associated with Complex Joint Phenomena in Multibody System Dynamics



T. Slimak, A. Zwölfer, F. Trainotti, D. Rixen
[tomas.slimak, andreas.zwoelfer, francesco.trainotti, rixen]@tum.de

Paper ID 65, July 25th, 2023

The Team

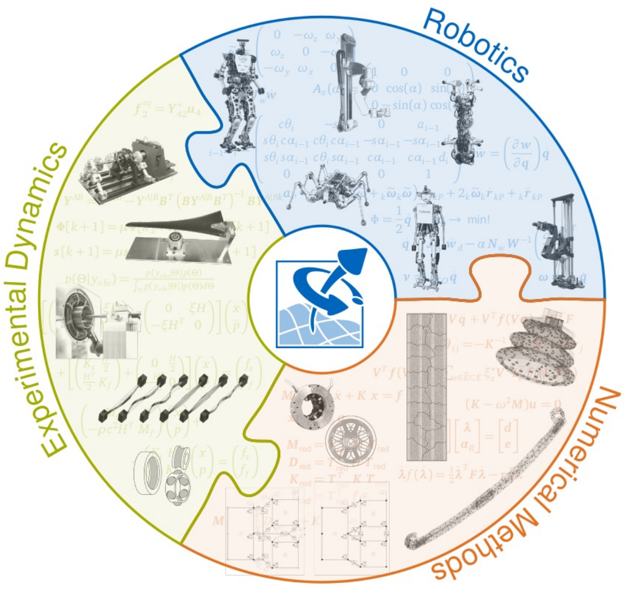
T. Slimak

A. Zwölfer

F. Trainotti

D. Rixen

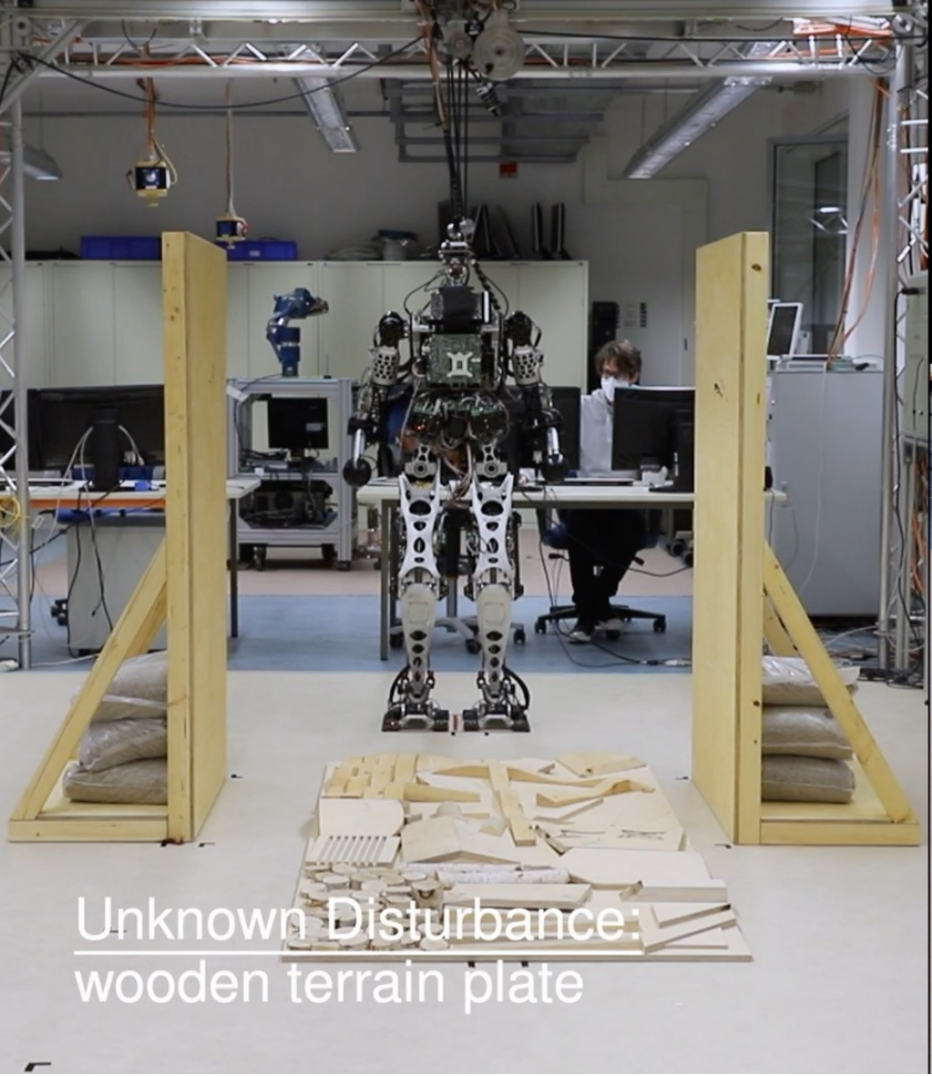
Research @ TUM-AM

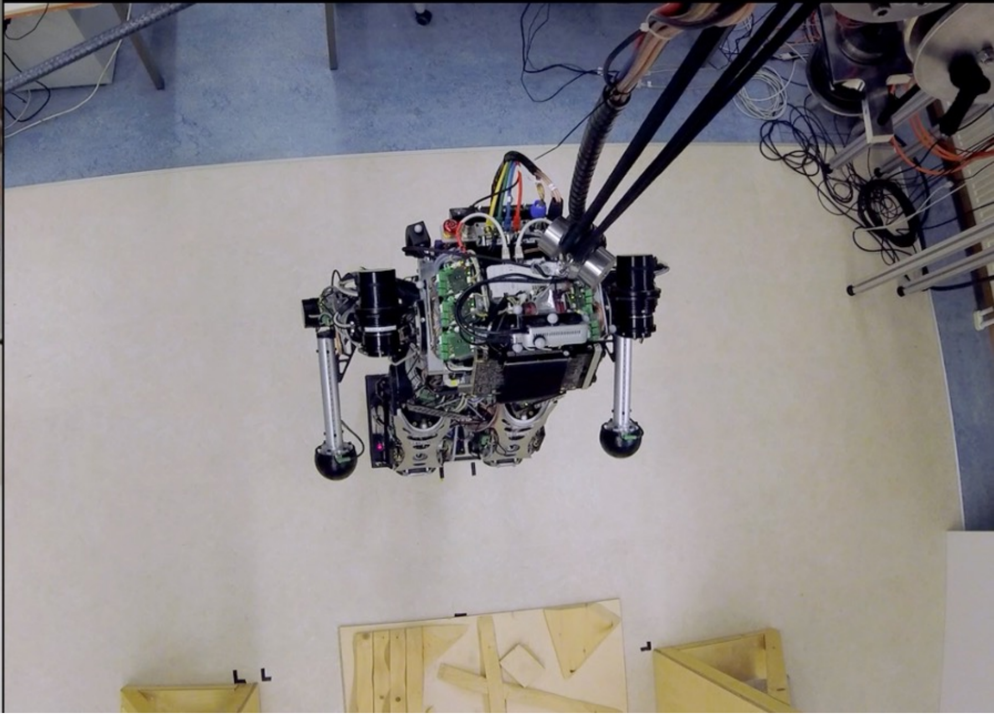


"[…] **Experimental** and **Numerical Methods** for Efficient Mechanical System Dynamics Simulations, […] Build Novel **Robotic** Systems."

- ❑ Model Reduction
- ❑ Substructuring
- ❑ Data-Driven Reduction & Dynamics
- ❑ Joint Dynamics
- ❑ Flexible Multibody Dynamics
- ❑ Steady State Solution Techniques
- ❑ Humanoid Robots

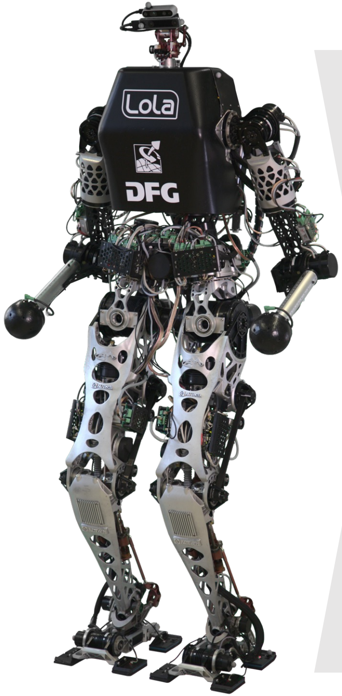
Video of *LOLA*

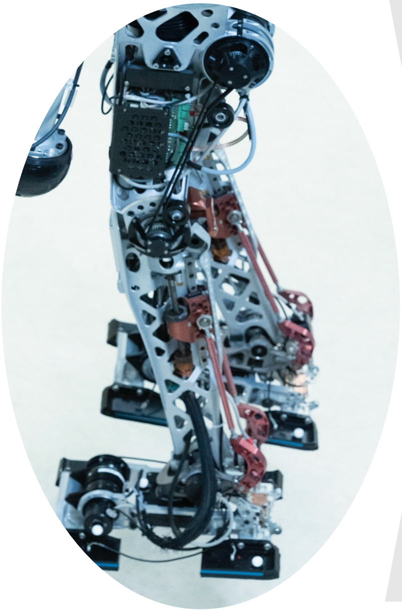


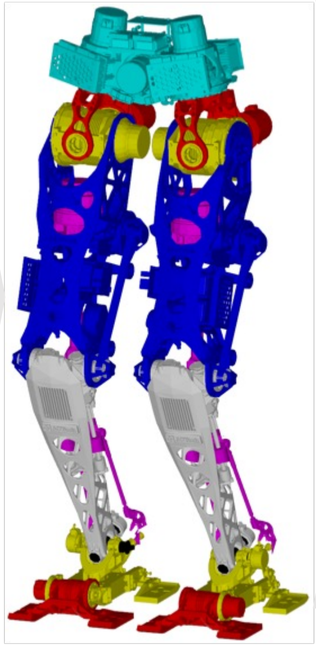


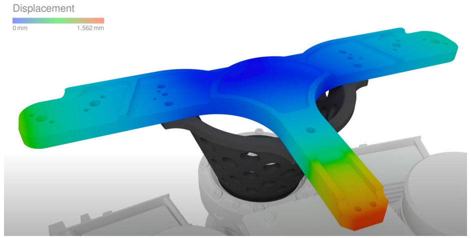
Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

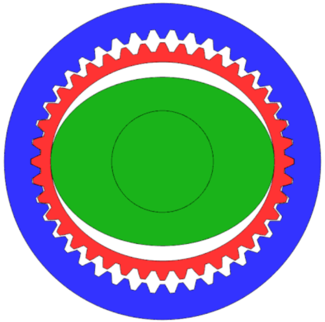
AM’s Humanoid Robot *LOLA*





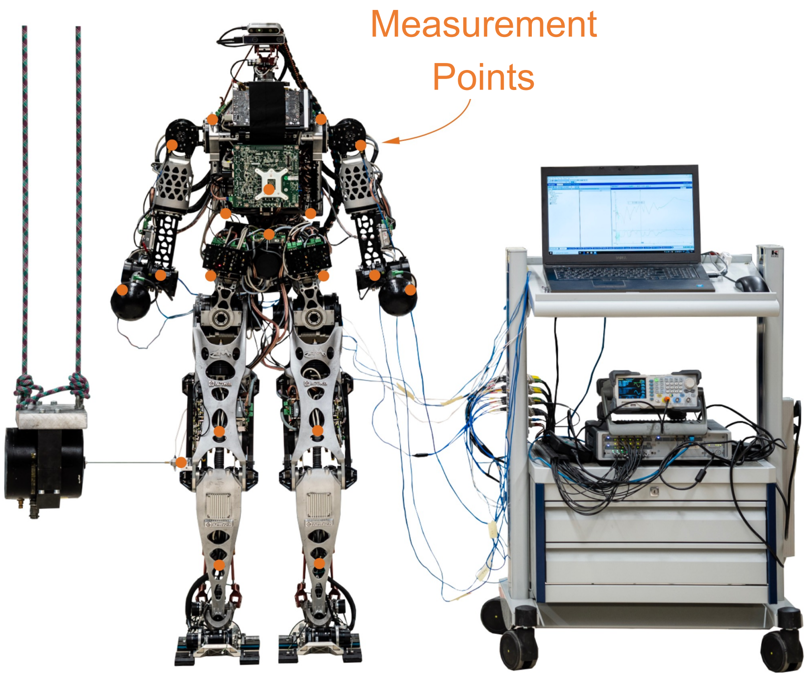


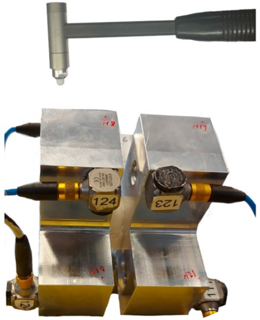


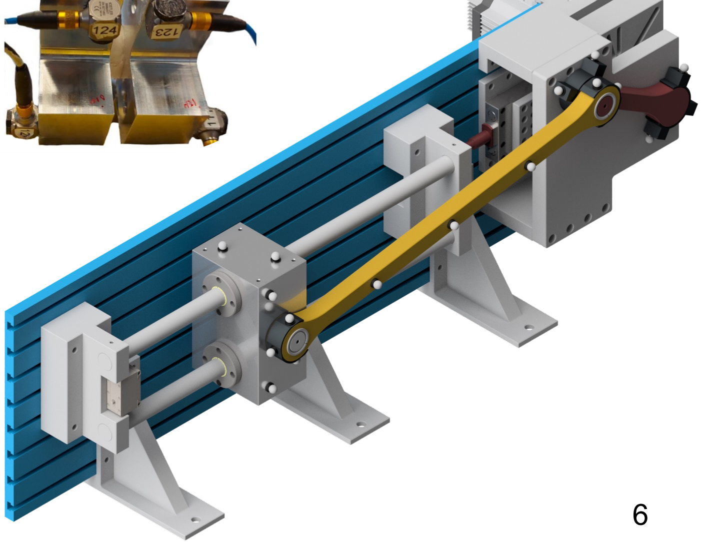


Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

Data From Experiments & Simulations







Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

Sparse Identification of Non-Linear Dynamics

$$\boldsymbol{M}(\boldsymbol{q})\,\ddot{\boldsymbol{q}}(t) = \boldsymbol{f}^{\mathrm{k}}(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t) + \boldsymbol{f}^{\mathrm{u}}(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t)$$

$$\boldsymbol{f}^{\mathrm{u}}(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t) \approx \boldsymbol{\Xi}^{\top}\boldsymbol{\lambda}(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t)$$

$$\boldsymbol{\lambda}(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t) = \begin{bmatrix} \lambda_1(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t) \\ \vdots \\ \lambda_L(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), t) \end{bmatrix} \overset{\text{e.g.}}{=} \begin{bmatrix} 1 \\ \boldsymbol{q}(t)^{\circ n} \\ \dot{\boldsymbol{q}}(t)^{\circ n} \\ \sin(\boldsymbol{q}(t)) \\ \operatorname{sgn}(\dot{\boldsymbol{q}}(t)) \\ \vdots \end{bmatrix}$$

Sparse Identification of Non-Linear Dynamics

$$\boldsymbol{Q} = \left[\boldsymbol{q}\left(t_1\right) \quad \ldots \quad \boldsymbol{q}\left(t_M\right)\right]$$

$$\dot{\boldsymbol{Q}} = \left[\dot{\boldsymbol{q}}\left(t_1\right) \quad \ldots \quad \dot{\boldsymbol{q}}\left(t_M\right)\right]$$

$$\ddot{\boldsymbol{Q}} = \left[\ddot{\boldsymbol{q}}\left(t_1\right) \quad \ldots \quad \ddot{\boldsymbol{q}}\left(t_M\right)\right]$$

$$\boldsymbol{\Lambda} = \left[\boldsymbol{\lambda}\left(\boldsymbol{q}\left(t_1\right), \dot{\boldsymbol{q}}\left(t_1\right), t_1\right) \quad \ldots \quad \boldsymbol{\lambda}\left(\boldsymbol{q}\left(t_M\right), \dot{\boldsymbol{q}}\left(t_M\right), t_M\right)\right]$$

$$\boldsymbol{\Xi} = \left[\boldsymbol{\xi}_1 \quad \ldots \quad \boldsymbol{\xi}_S\right]$$

$$\boldsymbol{M}\ddot{\boldsymbol{Q}} \approx \boldsymbol{F}^{\mathrm{k}} + \boldsymbol{\Xi}^{\top}\boldsymbol{\Lambda}$$

$$\boldsymbol{F}^{\mathrm{k}} = \left[\boldsymbol{f}^{\mathrm{k}}\left(\boldsymbol{q}\left(t_1\right), \dot{\boldsymbol{q}}\left(t_1\right), t_1\right) \quad \ldots \quad \boldsymbol{f}^{\mathrm{k}}\left(\boldsymbol{q}\left(t_M\right), \dot{\boldsymbol{q}}\left(t_M\right), t_M\right)\right]$$

Sequentially Thresholded Least Squares (STLS)

Algorithm 1 STLS pseudocode

Input: $\kappa, \boldsymbol{\Lambda}, \boldsymbol{F} = \boldsymbol{F}^{\mathrm{k}} - \boldsymbol{M}\ddot{\boldsymbol{Q}}$

Output: $\boldsymbol{\Xi}$

$\boldsymbol{\Xi} = \underset{\boldsymbol{\Xi} \in \mathbb{R}}{\arg\min} \left\| \boldsymbol{F}^{\top} - \boldsymbol{\Lambda}^{\top} \boldsymbol{\Xi} \right\|_2$

for iter $= 1, 2, 3, \ldots$ **do**

$\quad \boldsymbol{\Xi}\left(|\Xi_{ij}| < \kappa\right) \leftarrow 0$

$\quad$ **for** $c = 1, 2, \ldots, n_{\mathrm{c}}$ **do**

$\quad\quad \mathrm{col}_c \boldsymbol{\Xi}\left(|\Xi_{ij}| \geq \kappa\right) \leftarrow \underset{\boldsymbol{\Xi} \in \mathbb{R}}{\arg\min} \left\| \mathrm{col}_c \boldsymbol{F}^{\top} - \boldsymbol{\Lambda}_*^{\top} \mathrm{col}_c \boldsymbol{\Xi}\left(|\Xi_{ij}| \geq \kappa\right) \right\|_2$

Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

Simulated Example: Duffing Oscillator

$$f^{\mathrm{k}} = 0$$

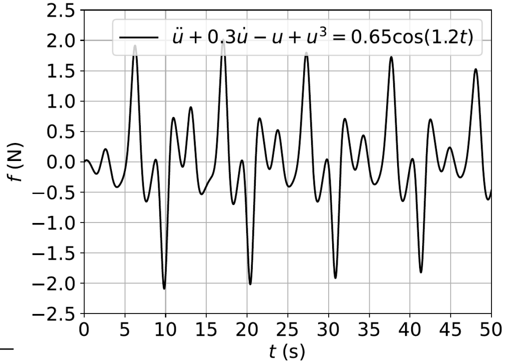
u	$\dot{u}$	u^2	u^3	1	$\mathrm{sgn}(\dot{u})$	$\cos(\omega t)$	$\cos(3\omega t)$	$u\dot{u}$
1	-0.3	0	-1	0	0	0.65	0	0

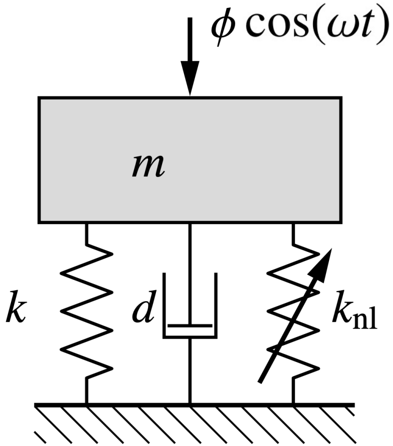
$$f^{\mathrm{k}} = 0.65\cos(1.2t)$$

u	$\dot{u}$	u^2	u^3	1	$\mathrm{sgn}(\dot{u})$	$\cos(\omega t)$	$\cos(3\omega t)$	$u\dot{u}$
1	-0.3	0	-1	0	0	0	0	0

$$f^{\mathrm{k}} = 0.65\cos(1.2t) + u - 0.3\dot{u}$$

u	$\dot{u}$	u^2	u^3	1	$\mathrm{sign}(\dot{u})$	$\cos(\omega t)$	$\cos(3\omega t)$	$u\dot{u}$
0	0	0	-1	0	0	0	0	0





$$\lambda = \begin{bmatrix} u \\ \dot{u} \\ u^2 \\ u^3 \\ 1 \\ \mathrm{sgn}(\dot{u}) \\ \cos(1.2t) \\ \cos(3.6t) \\ u\dot{u} \end{bmatrix}$$

$$m\ddot{u} + d\dot{u} + ku + k_{\mathrm{nl}}u^3 = \phi\cos(\omega t)$$

Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

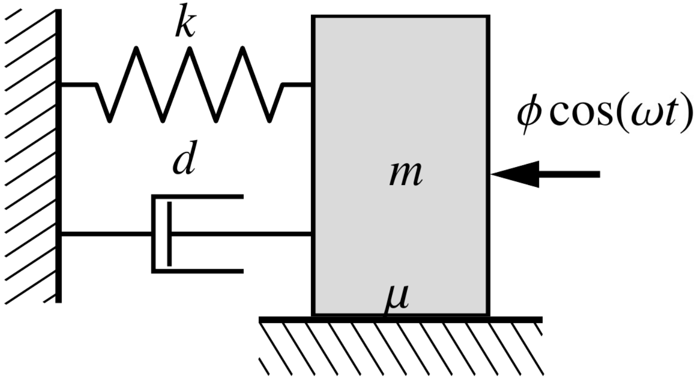
Simulated Example: Friction Slider

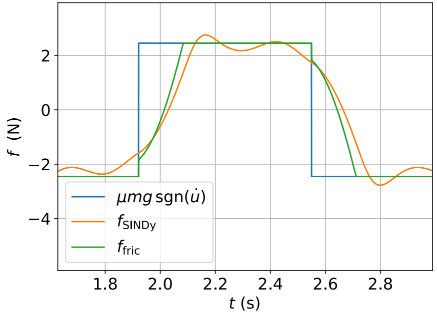
$$f^{\mathrm{k}} = 0$$

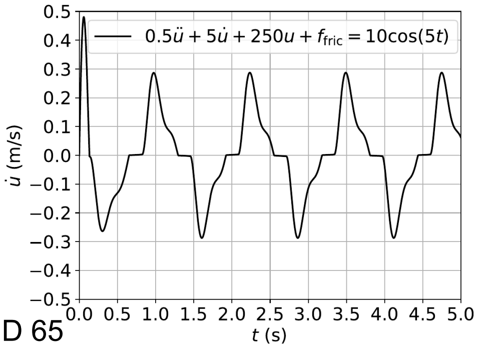
u	$\dot{u}$	u^2	u^3	1	$\mathrm{sgn}(\dot{u})$	$\cos(\omega t)$	$\cos(3\omega t)$	$u\dot{u}$	$f_{\mathrm{fric}}(u, \dot{u})$
-250	-5	0	0	0	0	10	0	0	-1

$$f^{\mathrm{k}} = 10\cos(5t) - ku - d\dot{u}$$

u	$\dot{u}$	u^2	u^3	1	$\mathrm{sign}(\dot{u})$	$\cos(\omega t)$	$\cos(3\omega t)$	$u\dot{u}$
136	0	-72	23231	0	0	-7	0	-12





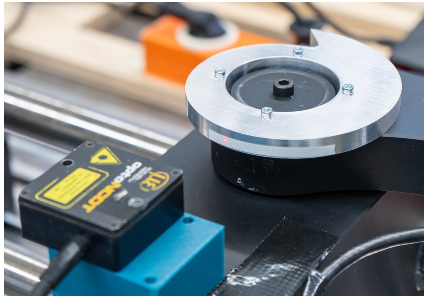


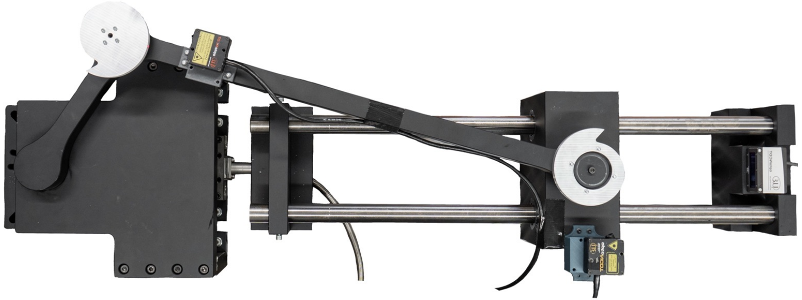
$$m\ddot{u} + d\dot{u} + ku + f_{\mathrm{fric}}(u, \dot{u}) = \phi\cos(\omega t)$$

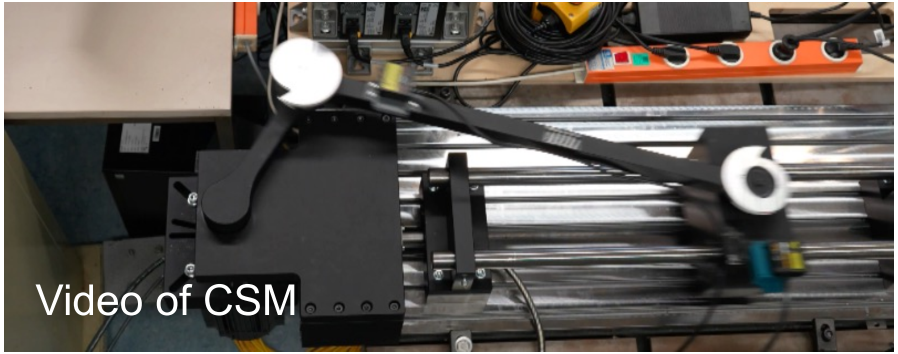
$$f_{\mathrm{fric}} = \begin{cases} \mu m g\,\mathrm{sgn}(\dot{u}) & \text{for slip} \\ \pm\mu m g + k_{\mu}\left(u - u_{\mathrm{csp}}\right) + d_{\mu}\dot{u} & \text{for stick} \end{cases}$$

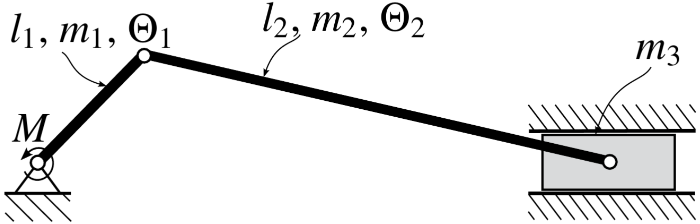
Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

Experimental Example: Crank-Shaft Mechanism



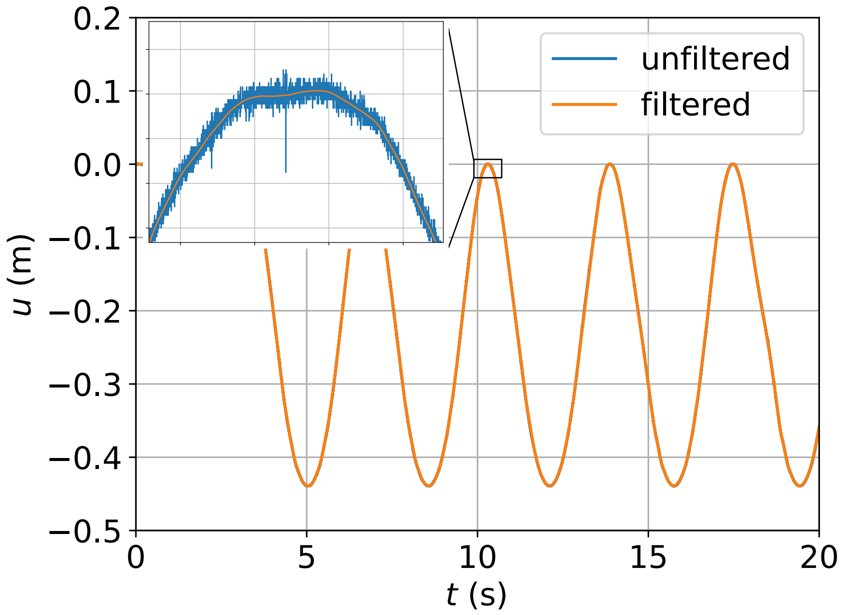


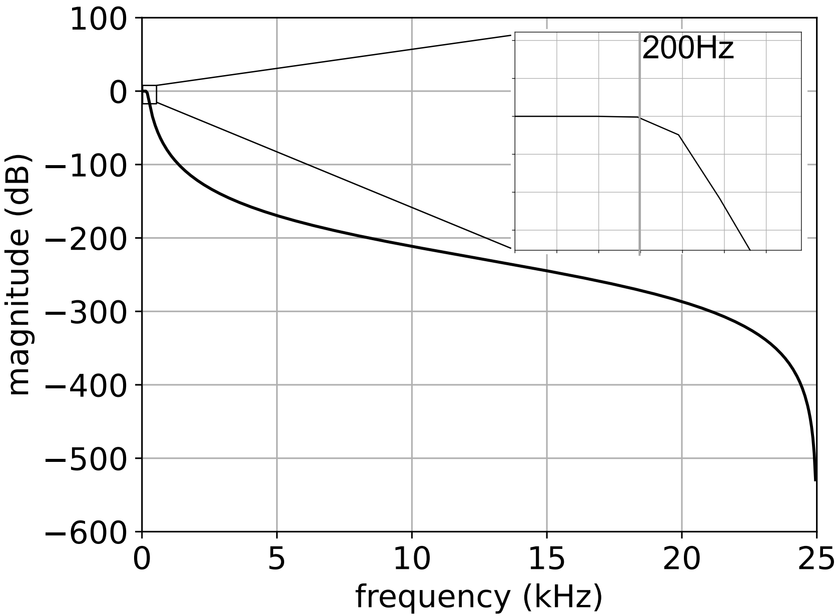




Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

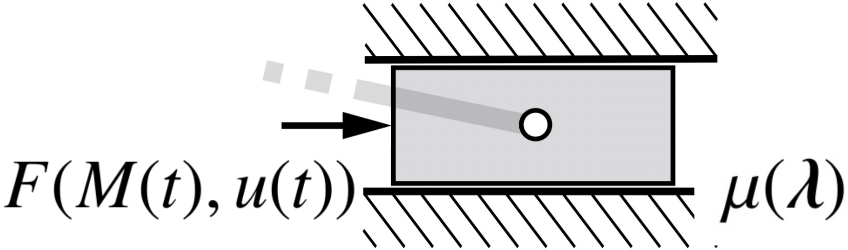
Experimental Example: Crank-Shaft Mechanism





Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

Experimental Example: Crank-Shaft Mechanism

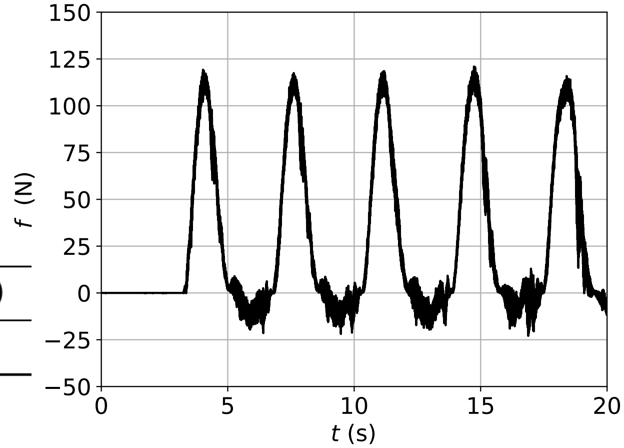


Experimental Data:

u	$\dot{u}$	u^2	u^3	1	$\mathrm{sgn}(\dot{u})$	$\sin(\omega t)$	$u\dot{u}$	$F(t)$
85	0	548	759	0	0	0	0	0

Simulation Data:

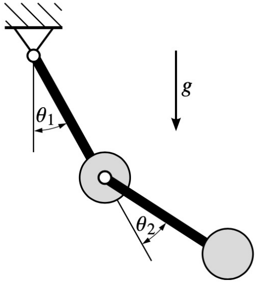
u	$\dot{u}$	u^2	u^3	1	$\mathrm{sgn}(\dot{u})$	$\sin(\omega t)$	$u\dot{u}$	$F(t)$
350	0	1632	2242	21	-19	0	0	1



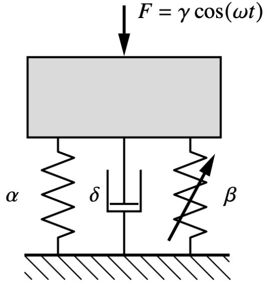
Take Home Messages

- Data-driven identification of analytical expressions for unknown forces is possible
- Works reliably with numerical data, real-world application is more difficult
- Good $\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}$ and excitation measurements are paramount

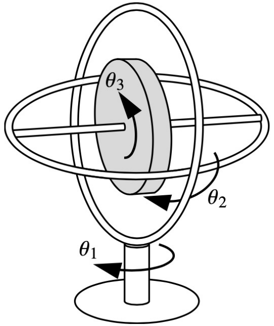
ASME Teaser: Neural Networks for MBS



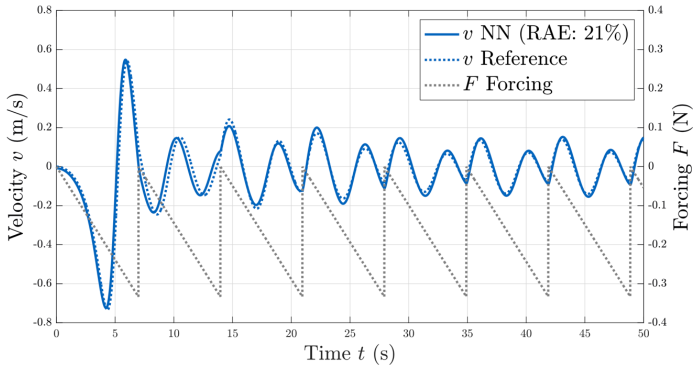
(a) Double pendulum

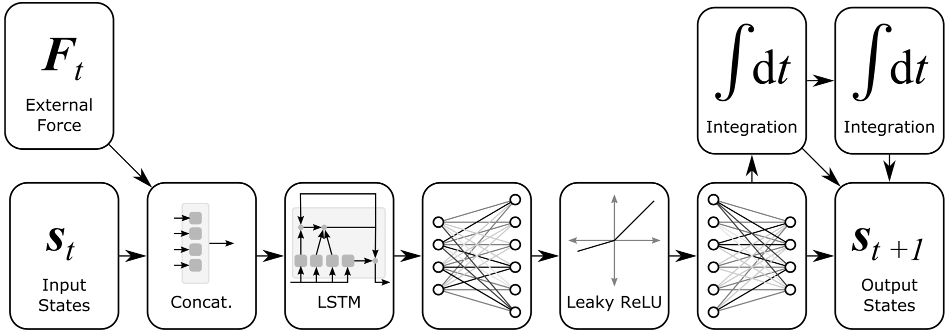


(b) Duffing oscillator



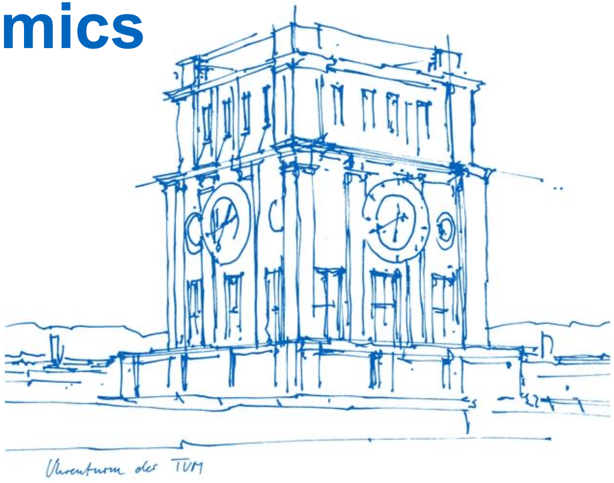
(c) 3D Gyroscope





Technical University of Munich
TUM School of Engineering and Design
Chair of Applied Mechanics

Sparse Identification of Unknown Equation of Motion Terms Associated with Complex Joint Phenomena in Multibody System Dynamics



T. Slimak, A. Zwölfer, F. Trainotti, D. Rixen
[tomas.slimak, andreas.zwoelfer, francesco.trainotti, rixen]@tum.de

Paper ID 65, July 25th 2023

Extra: Sequentially Thresholded Least Squares (STLS)

```
import numy as np
from numpy.linalg import lstsq

Xi = lstsq(Lambda.T,F.T,rcond=None)[0]
for k in range(10):
    smallInds = abs(Xi)<kappa
    Xi[smallInds] = 0
    for i in range(np.size(F,0)):
        bigInds = np.invert(smallinds[:,i])
        Xi[bigInds,i] = lstsq(Lambda.T[:,bigInds],F.T[:,i])[0]
```