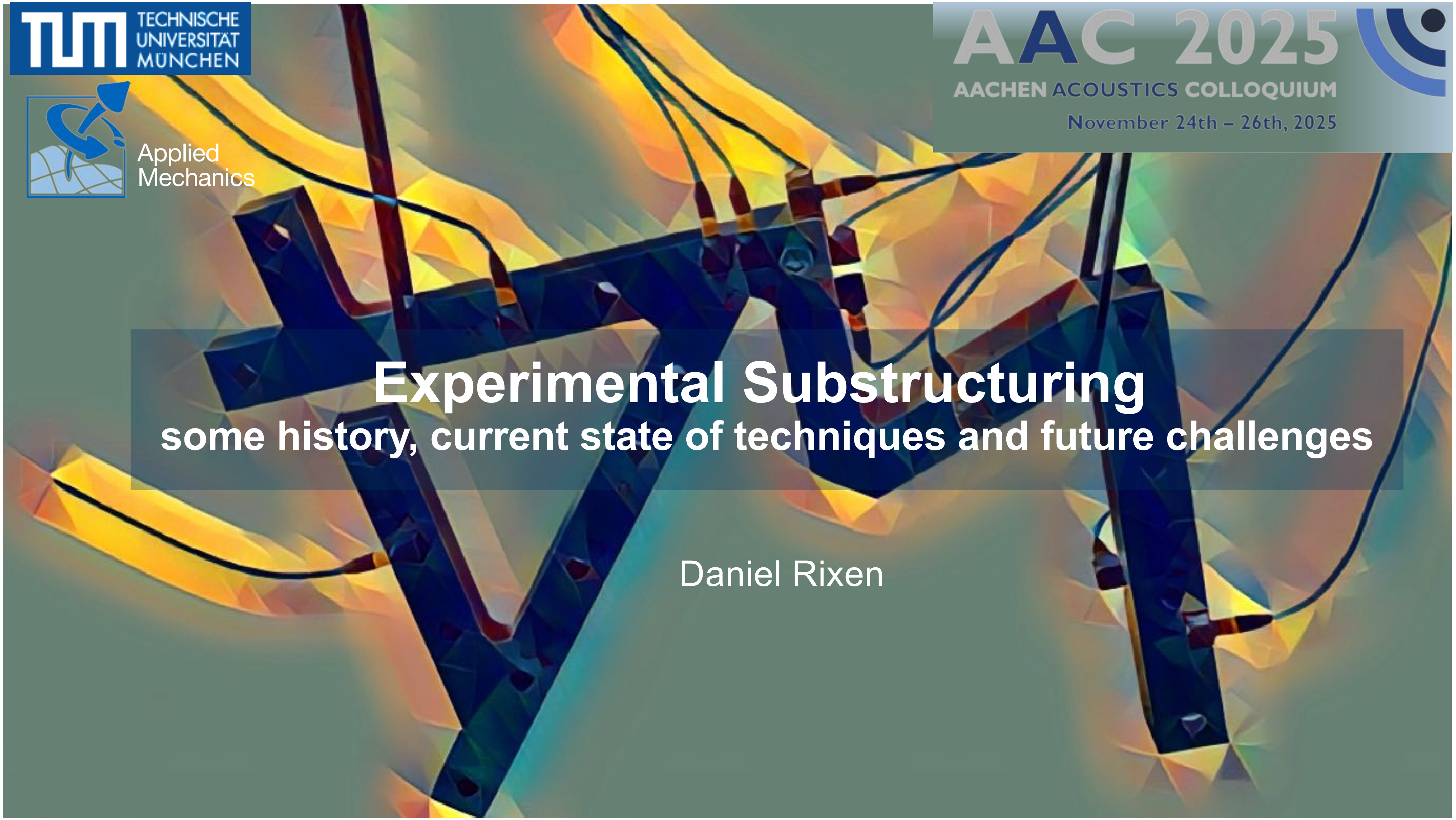
AAC 2025
AACHEN ACOUSTICS COLLOQUIUM
November 24th – 26th, 2025

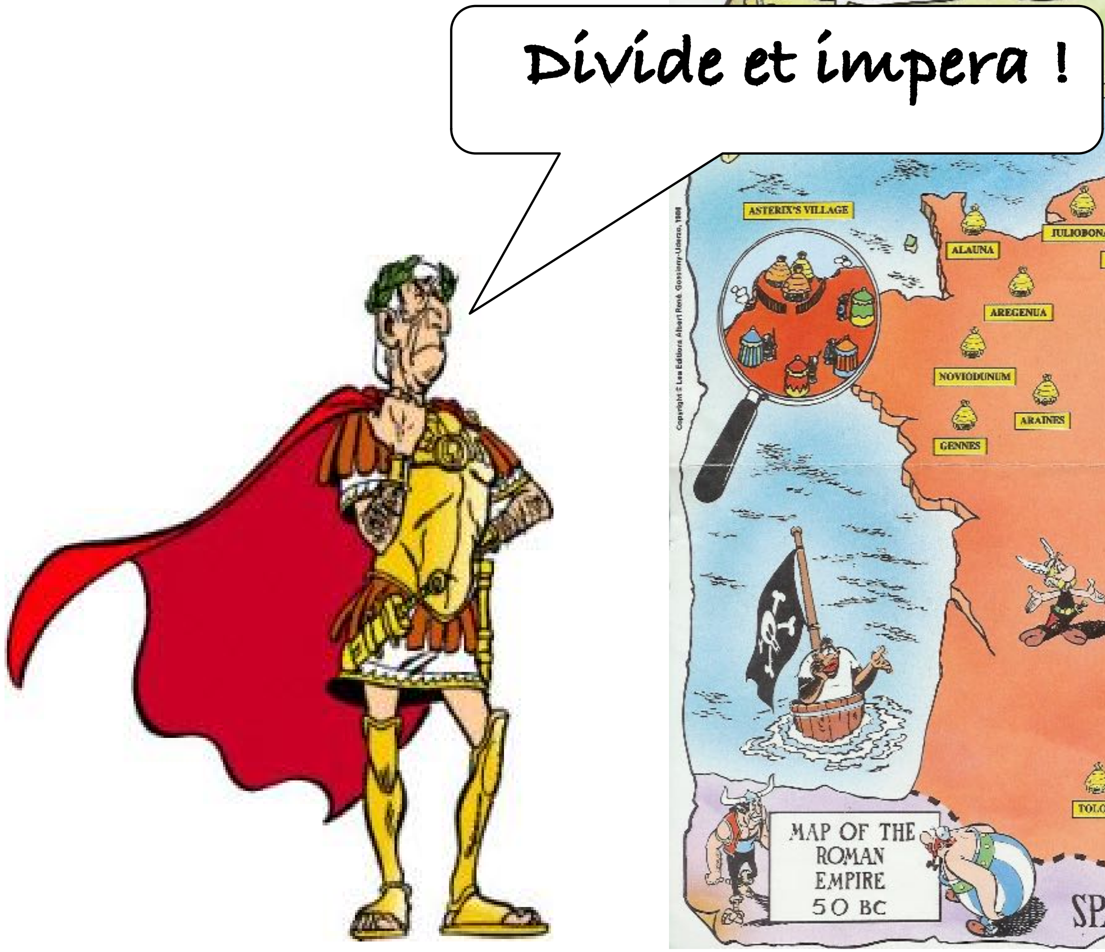
TECHNISCHE UNIVERSITÄT MÜNCHEN
Applied Mechanics

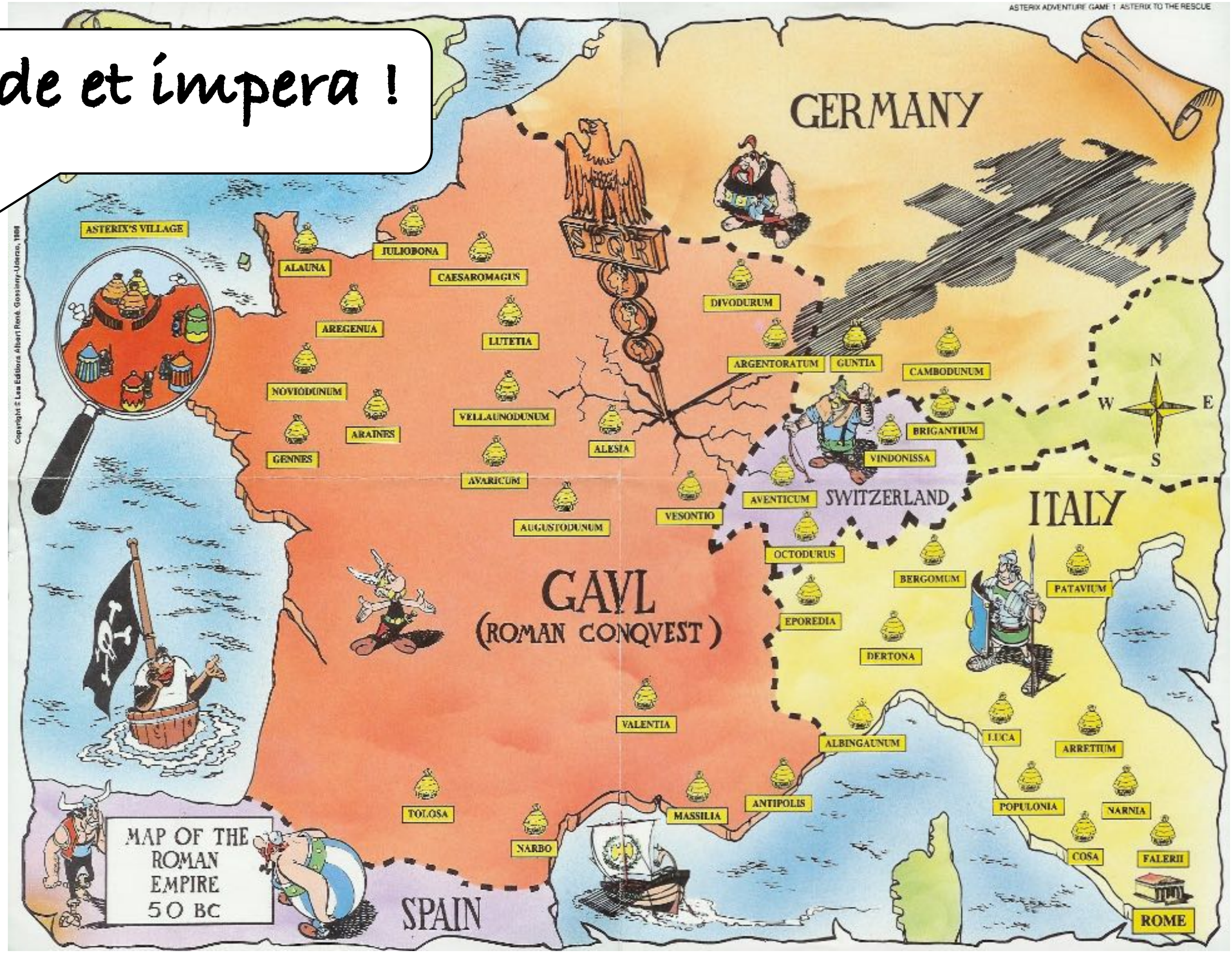


Experimental Substructuring

some history, current state of techniques and future challenges

Daniel Rixen


Divide et impera !


GERMANY
GAVL
(ROMAN CONQVEST)
SWITZERLAND
ITALY
SPAIN
MAP OF THE ROMAN EMPIRE 50 BC
SPQR
ASTERIX'S VILLAGE
ALAUNA
IULIOBONA
CAESAROMAGUS
AREGENUA
LUTETIA
NOVIODUNUM
ARAINES
GENNES
VELLAUNODUNUM
ALESIA
AVARICUM
AUGUSTODUNUM
VESONTIO
DIVODURUM
ARGENTORATUM
GUNTIA
CAMBODUNUM
BRIGANTIUM
VINDONISSA
AVENTICUM
OCTODURUS
BERGOMUM
PATAVIUM
EPOREDIA
DERTONA
VALENTIA
ALBINGAUNUM
LUCA
ARRETIUM
POPULONIA
NARNIA
COSA
FALERII
ROME
TOLOSA
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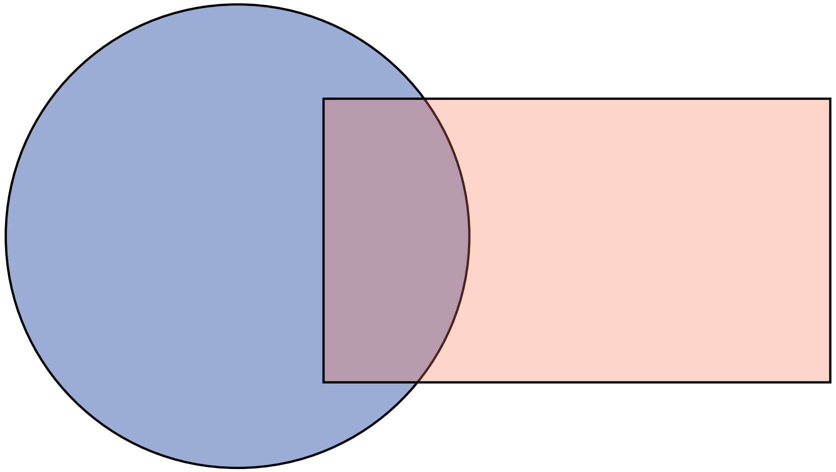
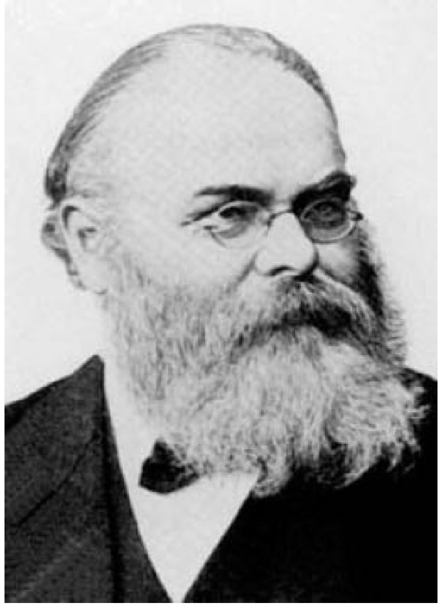
E

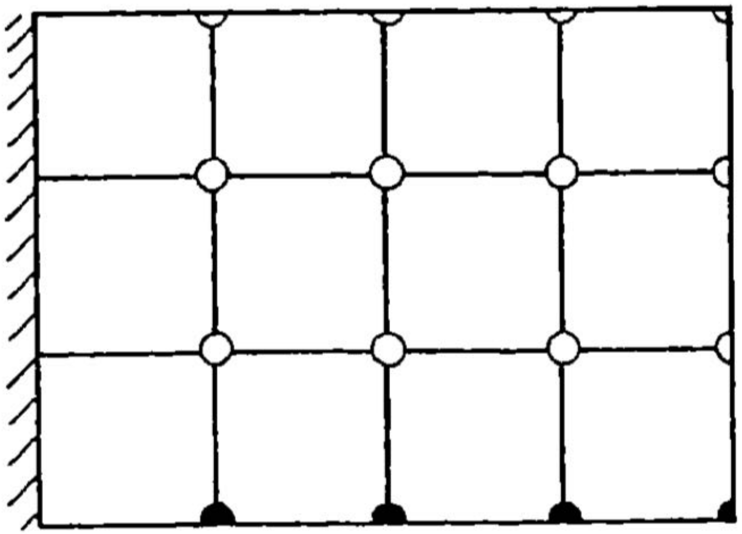
Then came the mathematicians

H. Schwarz, in 1890, wanted to prove the existence and uniqueness of the solution to the Laplace equation in a complex domain:



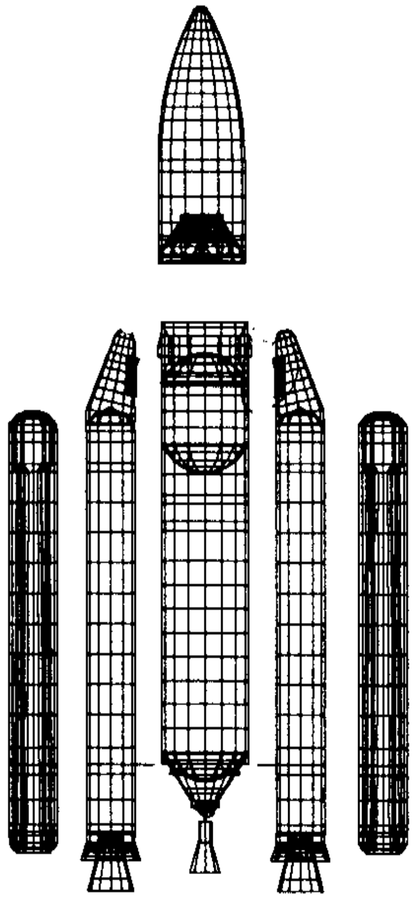


Richard Courant, in 1943, had the idea to subdivide domains into elements and use shape functions to approximate the solution in each elements. Finite Elements were born …

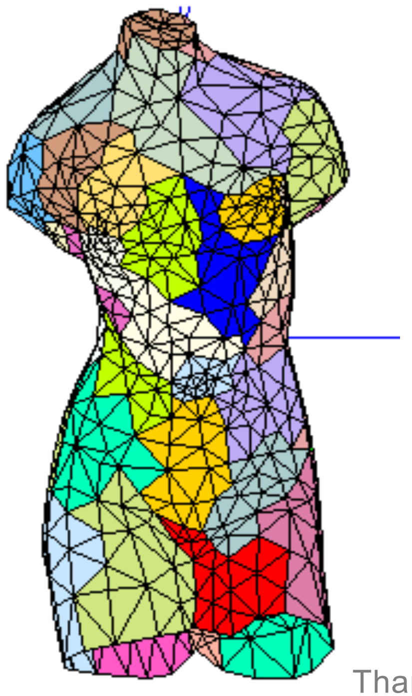


Then came the engineers!

In the 60's, the idea of partitioning a finite element model in substructures was proposed to reduce the complexity of the models (Gladwell, Hurty, Craig, Rubin, …)



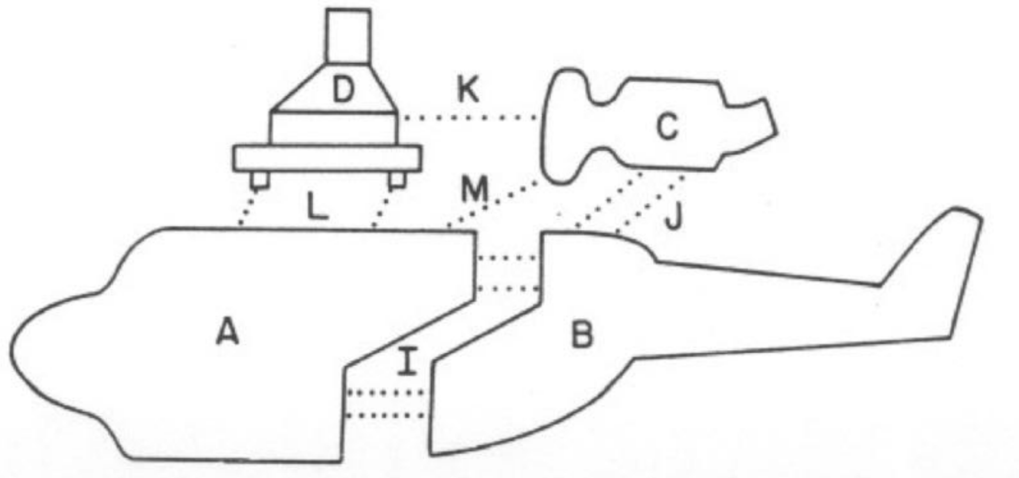
In the 80's, in order to speed-up the solution of linear algebraic system, the computational domain was cut in sub-domains to share work amongst several CPUs. That was the start of ***Domain Decomposition***.



Thanks to P. Gosselet

Decomposing system for **testing**
in the 70's (Klosterman, 71, SMURF),
in the 80's (Jetmundsen, 88)
and later in the 90's (Otte, 1991)

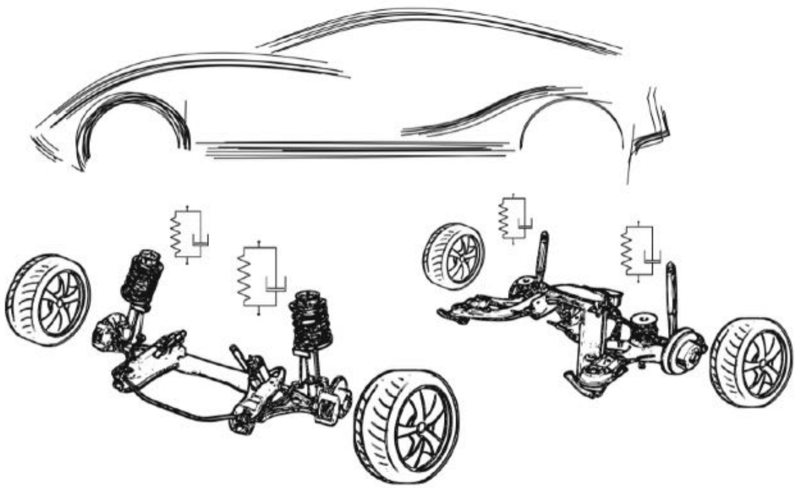
$$[Y'] = [Y_{\alpha\alpha}] - [[M] \oplus [Y_{\alpha\gamma}]] \cdot \left[\sum_{i=1}^{N} M_i^T M_i \oplus [Y_{\gamma\gamma}^i] \right]^{-1} [[M] \oplus [Y_{\alpha\gamma}]]^T$$

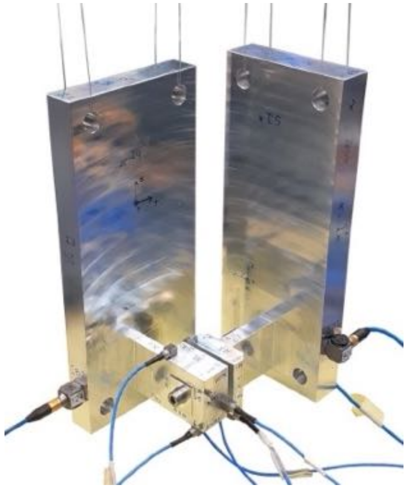


[Jetmundsen et al., 88]

Starting in 2000, **experimental substructuring** was then significantly improved thanks to

- *Cheaper and better sensors*
- *Cheaper multi-channels acquisition systems*
- *Better understanding of the effect of measurement errors and advanced algorithms for coupling*





...

- **Based on work done over the last 20 years with**

Dennis de Klerk
Sven Voormeeren
Paul vd Valk
Marteen vdSeijs
Michael Häußler
Steven Klaassen
Ahmed El Mahmoudi
Michael Kreutz
Francesco Trainotti
Verena Gimpl
Mert Göldeli
Oliver Zobel


Not disclosed

Contents

- **Motivation and some basics on dynamic matrices and assembly**
- **Frequency-Based Substructuring (FBS)**
  - Dual assembly of components (example of a guitar)
  - Weakening of the interface compatibility: virtual point transformation (example of AM structure)
- **Decoupling**
  - Transmission simulator
  - Joint identification
- **The concept of blocked forces**
- **Things not discussed, but certainly interesting …**
- **Current research directions**

Motivation: why experimental substructuring ?

- what sources create the excitation ?
- how do those sources propagate through the structure ("*transfer path*") to the receiver ?

101 on dynamic stiffness

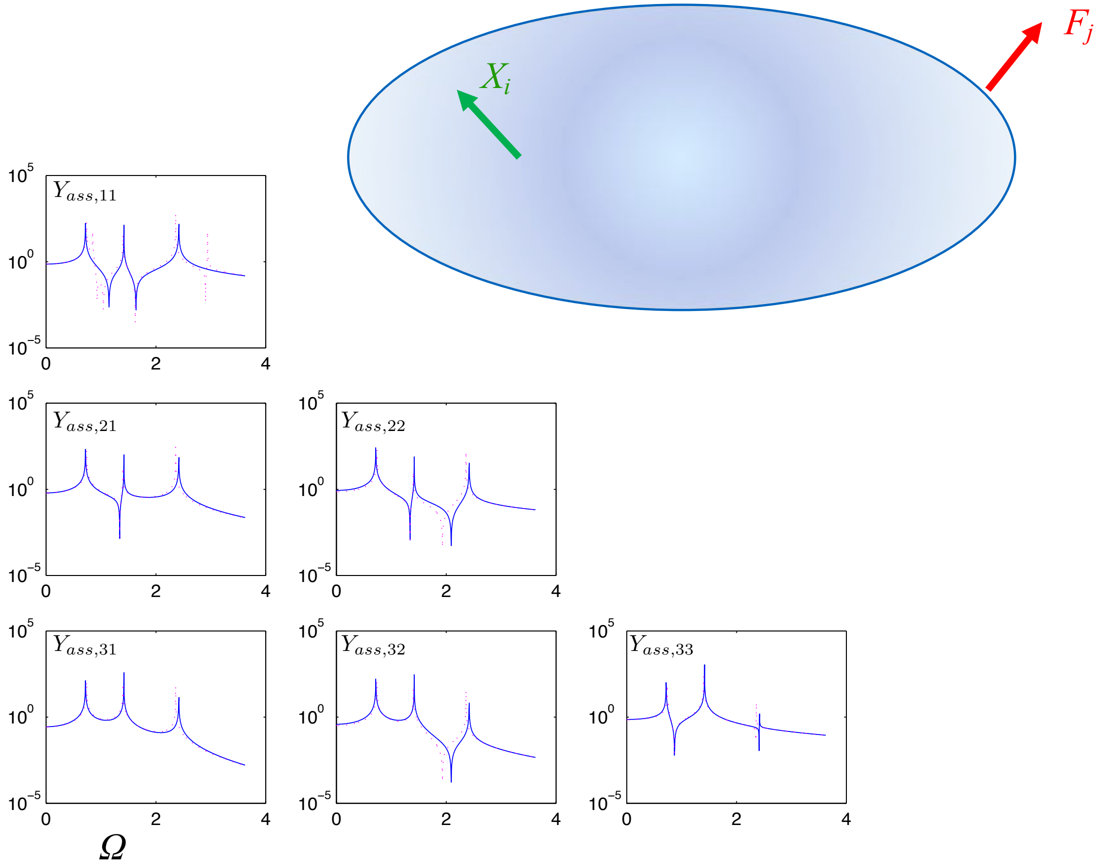
A structure can be characterized by its impedance or admittance between any pair of input and output:

$$(\boldsymbol{K} - \Omega^2 \boldsymbol{M})\boldsymbol{X} = \boldsymbol{F}$$

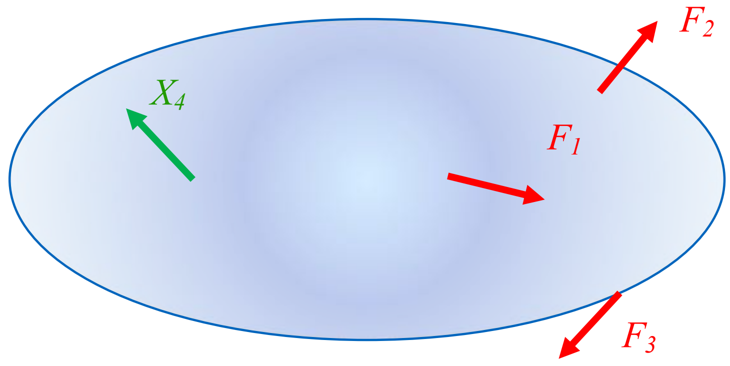
dynamic stiffness
"Impedance"

$$\boldsymbol{X} = (\boldsymbol{K} - \Omega^2 \boldsymbol{M})^{-1}\boldsymbol{F} = \boldsymbol{Y}\boldsymbol{F}$$

dynamic flexibility
"Admittance" / FRF
=
what is measured in practice



Consider 3 sources of excitations (F_1, F_2, F_3)
and vibration level X_4 at location 4.



$$\boldsymbol{X} = \boldsymbol{Y}\boldsymbol{F} \longrightarrow x_4 = \begin{bmatrix} Y_{41} & Y_{42} & Y_{43} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ Y_3 \end{bmatrix} = Y_{41}F_1 + Y_{42}F_2 + Y_{43}F_3$$

- The contribution of each force and each path to the response can be evaluated
- With this understanding, one can change the vibration level or "design" specific vibration spectra at the output by
    - modifying the transfer paths Y_{41}, Y_{42}, Y_{43}
    - modifying the excitation sources F_1, F_2, F_3

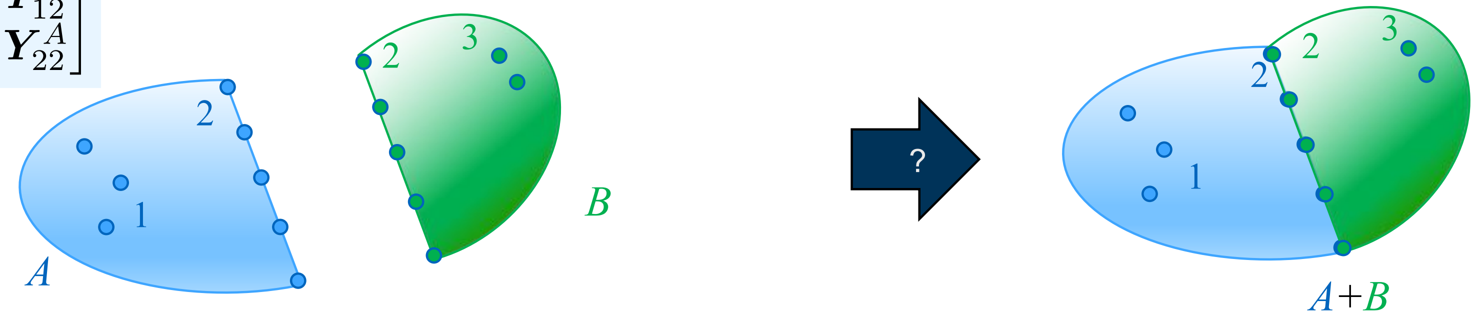
Dual assembly of components

Assume we know the dynamics of 2 components from their admittances obtained from a numerical model or from measurements:

$$Y^{A+B} = \begin{bmatrix} \boldsymbol{Y}_{11}^{A+B} & \boldsymbol{Y}_{12}^{A+B} & \boldsymbol{Y}_{13}^{A+B} \\ \boldsymbol{Y}_{21}^{A+B} & \boldsymbol{Y}_{22}^{A+B} & \boldsymbol{Y}_{23}^{A+B} \\ \boldsymbol{Y}_{31}^{A+B} & \boldsymbol{Y}_{32}^{A+B} & \boldsymbol{Y}_{33}^{A+B} \end{bmatrix}$$

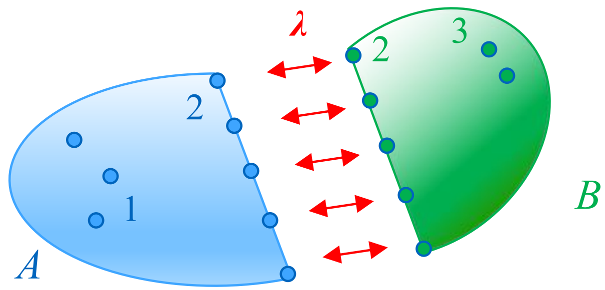
$$Y^{B} = \begin{bmatrix} \boldsymbol{Y}_{22}^{B} & \boldsymbol{Y}_{23}^{B} \\ \boldsymbol{Y}_{32}^{B} & \boldsymbol{Y}_{33}^{B} \end{bmatrix}$$

$$Y^{A} = \begin{bmatrix} \boldsymbol{Y}_{11}^{A} & \boldsymbol{Y}_{12}^{A} \\ \boldsymbol{Y}_{21}^{A} & \boldsymbol{Y}_{22}^{A} \end{bmatrix}$$



- $\boldsymbol{x}_1$ all internal dofs in substructure A (where later a force will be applied)
- $\boldsymbol{x}_2$ all interface dofs in substructure A, coinciding with all interface dofs in substructure B.
- $\boldsymbol{x}_3$ all internal dofs in substructure B (where later the output response will be analysed)

$$Y^A = \begin{bmatrix} \boldsymbol{Y}_{11}^A & \boldsymbol{Y}_{12}^A \\ \boldsymbol{Y}_{21}^A & \boldsymbol{Y}_{22}^A \end{bmatrix}$$



$$Y^B = \begin{bmatrix} \boldsymbol{Y}_{22}^B & \boldsymbol{Y}_{23}^B \\ \boldsymbol{Y}_{32}^B & \boldsymbol{Y}_{33}^B \end{bmatrix}$$

The dynamic equation of each substructure in the frequency domain when it is assembled writes

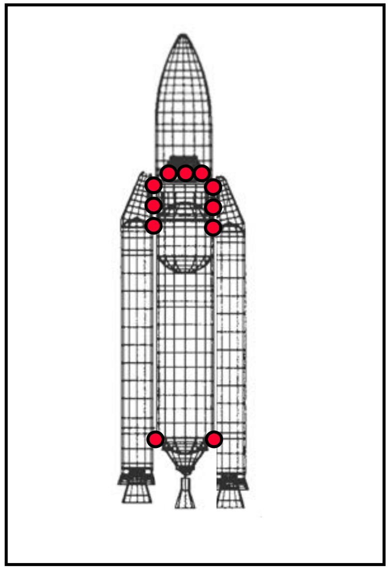
$$\begin{bmatrix} \boldsymbol{x}_1^A \\ \boldsymbol{x}_2^A \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{11}^A & \boldsymbol{Y}_{12}^A \\ \boldsymbol{Y}_{21}^A & \boldsymbol{Y}_{22}^A \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_1^A \\ \boldsymbol{\lambda} \end{bmatrix}$$
$$\begin{bmatrix} \boldsymbol{x}_2^B \\ \boldsymbol{x}_3^B \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{22}^B & \boldsymbol{Y}_{23}^B \\ \boldsymbol{Y}_{32}^B & \boldsymbol{Y}_{33}^B \end{bmatrix} \begin{bmatrix} -\boldsymbol{\lambda} \\ \boldsymbol{f}_3^B \end{bmatrix}$$

where the Lagrange multipliers $\boldsymbol{\lambda}$ are the internal forces ensuring interface compatibility:

$$\boldsymbol{x}_2^A - \boldsymbol{x}_2^B = 0$$

These sets of equations (dynamic equilibrium and compatibility) are called the *dually assembled form*.

101 on assembly: Primal / Dual Assembly

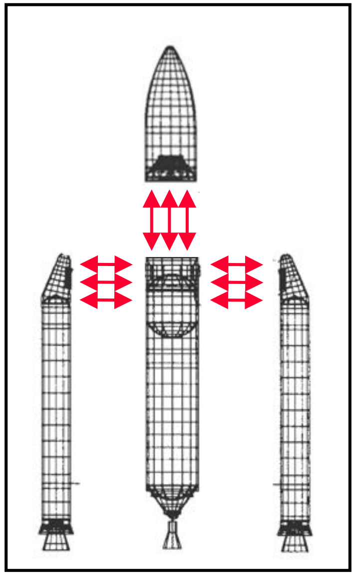


Mathematically equivalent

… but allows different approximations:

weaken
interface equilibrium

weaken
interface compatibility

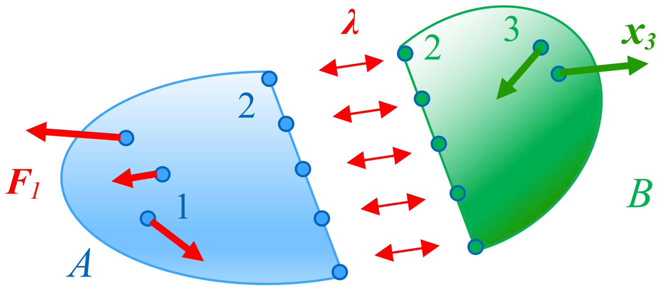


$$\begin{bmatrix} K^{(1)} & & 0 \\ & \ddots & \\ 0 & & K^{(N_s)} \end{bmatrix} \begin{bmatrix} u^{(1)} \\ \vdots \\ u^{(N_s)} \end{bmatrix} = \begin{bmatrix} f^{(1)} \\ \vdots \\ f^{(N_s)} \end{bmatrix}$$

signed Boolean matrices

$$\begin{bmatrix} K^{(1)} & & 0 & B^{(1)^T} \\ & \ddots & & \vdots \\ 0 & & K^{(N_s)} & B^{(N_s)^T} \\ B^{(1)} & \dots & B^{(N_s)} & 0 \end{bmatrix} \begin{bmatrix} u^{(1)} \\ \vdots \\ u^{(N_s)} \\ \lambda \end{bmatrix} = \begin{bmatrix} f^{(1)} \\ \vdots \\ f^{(N_s)} \\ 0 \end{bmatrix}$$

Assume forces on dofs 1 in A: response at dofs 3 in B ?



$$\begin{bmatrix} x_1^A \\ x_2^A \end{bmatrix} = \begin{bmatrix} Y_{11}^A & Y_{12}^A \\ Y_{21}^A & Y_{22}^A \end{bmatrix} \begin{bmatrix} f_1^A \\ \lambda \end{bmatrix}$$

$$\begin{bmatrix} x_2^B \\ x_3^B \end{bmatrix} = \begin{bmatrix} Y_{22}^B & Y_{23}^B \\ Y_{32}^B & Y_{33}^B \end{bmatrix} \begin{bmatrix} -\lambda \\ 0 \end{bmatrix}$$

Dually assembled form

$$x_2^A - x_2^B = 0$$

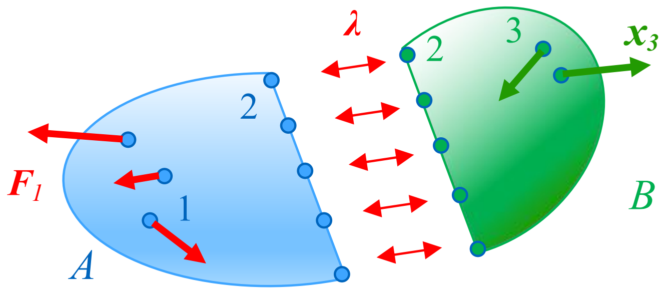
- From the dynamic equations of each substructure, we write the interface displacements as function of the interface forces:

$$\begin{aligned} x_2^A &= Y_{21}^A f_1^A + Y_{22}^A \lambda \\ x_2^B &= -Y_{22}^B \lambda \end{aligned}$$

- Replacing then in the interface compatibility condition, we find the so-called *dual interface problem*

$$(Y_{22}^A + Y_{22}^B)\lambda = -Y_{21}^A f_1^A$$

→ determines the amplitude of the interface forces needed to close the gap created by applied forces



$$\begin{bmatrix} \boldsymbol{x}_1^A \\ \boldsymbol{x}_2^A \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{11}^A & \boldsymbol{Y}_{12}^A \\ \boldsymbol{Y}_{21}^A & \boldsymbol{Y}_{22}^A \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_1^A \\ \boldsymbol{\lambda} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{x}_2^B \\ \boldsymbol{x}_3^B \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{22}^B & \boldsymbol{Y}_{23}^B \\ \boldsymbol{Y}_{32}^B & \boldsymbol{Y}_{33}^B \end{bmatrix} \begin{bmatrix} -\boldsymbol{\lambda} \\ 0 \end{bmatrix}$$

Dually assembled form

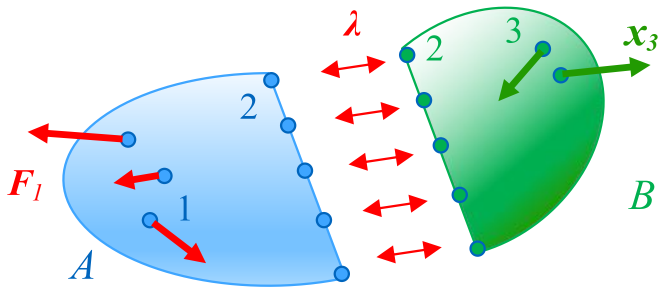
$$\boldsymbol{x}_2^A - \boldsymbol{x}_2^B = 0$$

Solving then for the interface forces

$$(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)\boldsymbol{\lambda} = -\boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A \longrightarrow \boldsymbol{\lambda} = -(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}\boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$$

and replacing in the dynamic response at the output 3 in substructure B:

$$\boldsymbol{x}_3^B = -\boldsymbol{Y}_{32}^B \boldsymbol{\lambda} \longrightarrow \boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}\boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$$



$$\boldsymbol{x}_3^B = \overbrace{\boldsymbol{Y}_{32}^B (\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1} \boldsymbol{Y}_{21}^A}^{Y_{31}^{A+B}} \boldsymbol{f}_1^A$$

Frequency-Based Substructuring (FBS)

Gap in the interface due to applied forces, when no interface force is present ($\boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$)

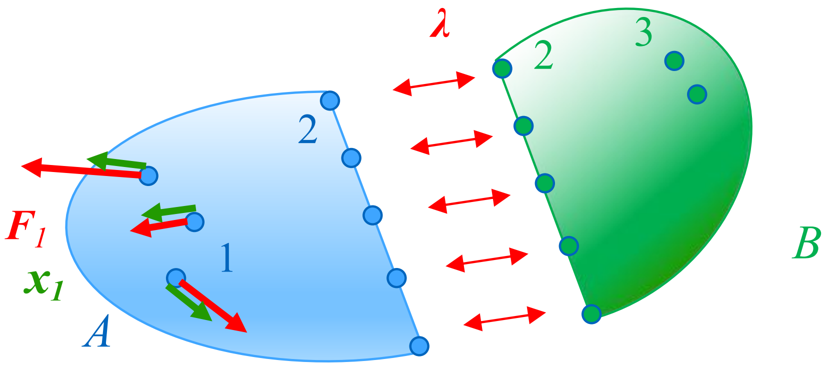
Internal interface forces $\boldsymbol{\lambda}$ to close the gap ($(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1} \boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$)

Remarks:

If the interface admittances $\boldsymbol{Y}_{22}^A, \boldsymbol{Y}_{22}^B$ are obtained from measurements, they will always be noisy and slightly wrong. Those errors will usually be strongly amplified when computing the inverse $(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}$ This is one of the biggest challenges in experimental substructuring.

Check yourself

In case one would be interested in the vibration response observed in the assembled system on dofs 1 (in the same substructure as where the forces are applied), the formula of the Frequency-Based Substructuring (FBS) would be:



a. $\boldsymbol{x}_1^A = \boldsymbol{Y}_{11}^A \boldsymbol{f}_1^A$

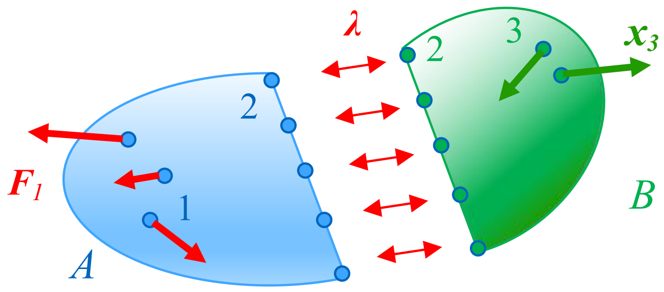
b. $\boldsymbol{x}_1^A = \boldsymbol{Y}_{12}^A (\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1} \boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$

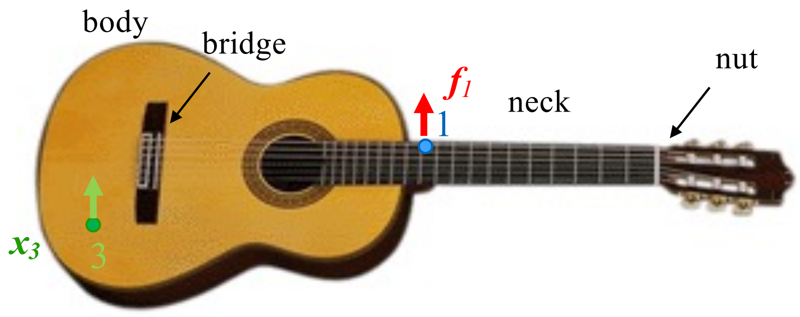
c. $\boldsymbol{x}_1^A = \left(\boldsymbol{Y}_{11}^A - \boldsymbol{Y}_{12}^A (\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1} \boldsymbol{Y}_{21}^A\right) \boldsymbol{f}_1^A$

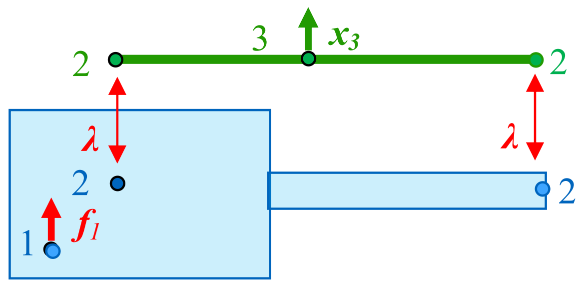
d. $\boldsymbol{x}_1^A = \boldsymbol{Y}_{12}^A (\boldsymbol{Y}_{22}^A)^{-1} \boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$

$$\boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B (\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1} \boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$$

Example of a guitar







Investigate the timber of a guitar :
(idea to do so came from the “Wolf Tone” of cellos)

Difficult to precisely apply a force on the string
→ force is given on the body and the output vibration is at the string.

Assumptions

- Interface at bridge and nut
- in-plane vibrations of the sound board and bridge assumed irrelevant

[1] D. Rixen, T. Godeby, and E. Pagnacco. Dual assembly of substructures and the fbs method: Application to the dynamic testing of a guitar. In *International Conference on Noise and Vibration Engineering, ISMA*, Leuven, Belgium, September 18-20 2006. KUL.

AM
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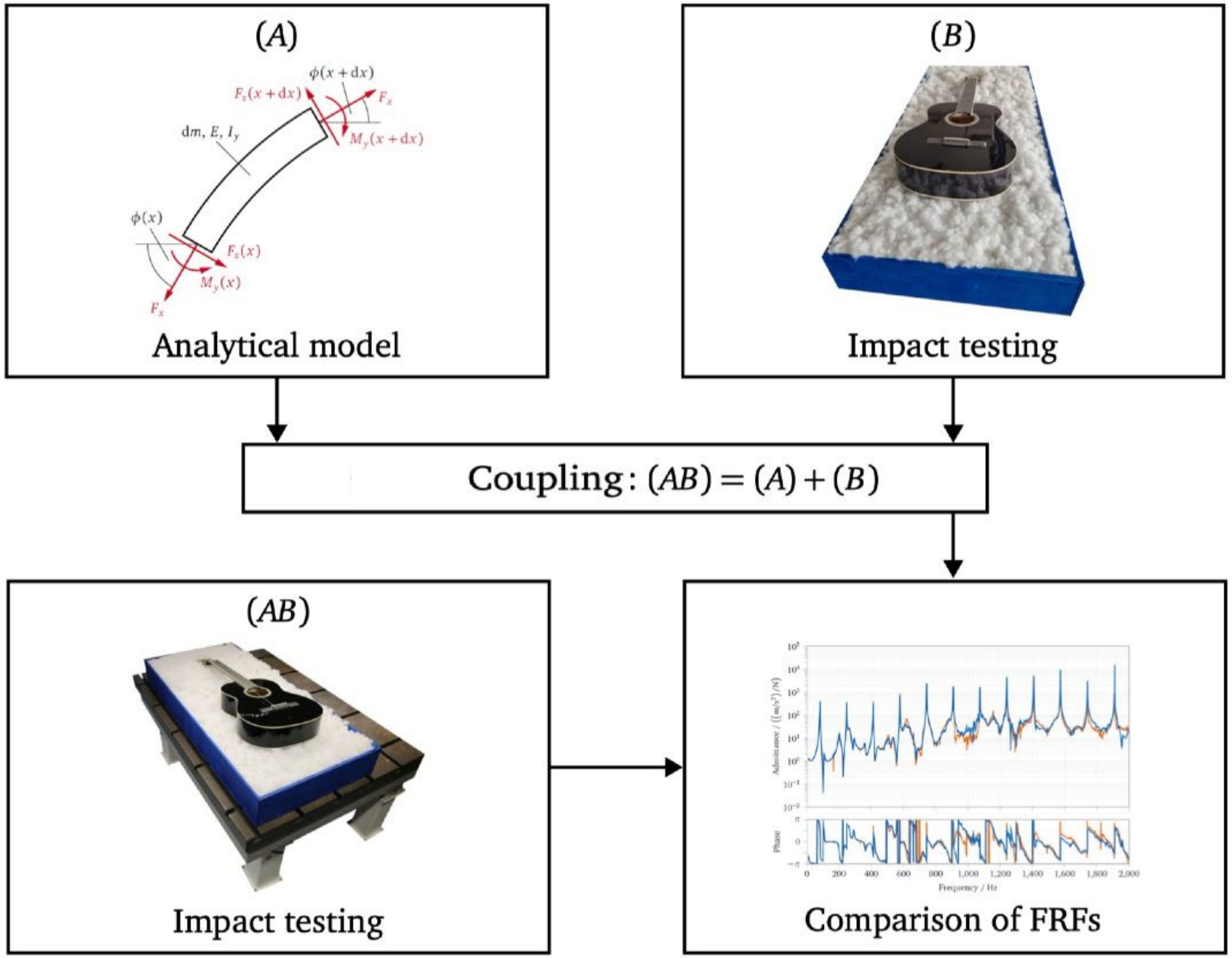
Angewandte Mechanik

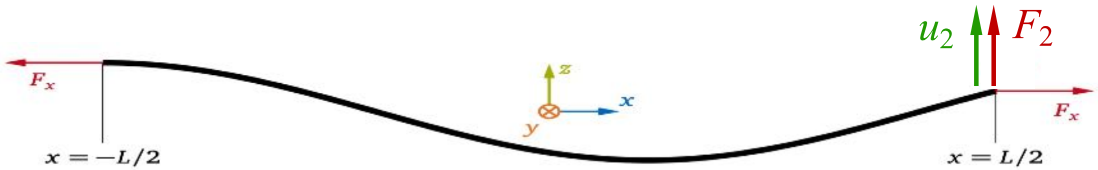


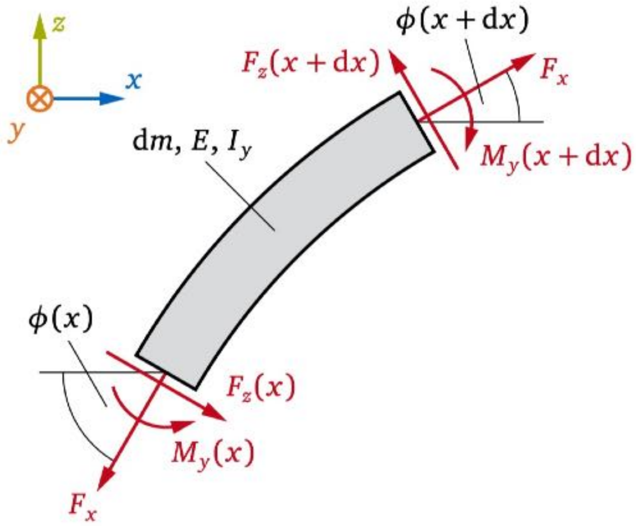
From [1]

Updated results from Student Project of

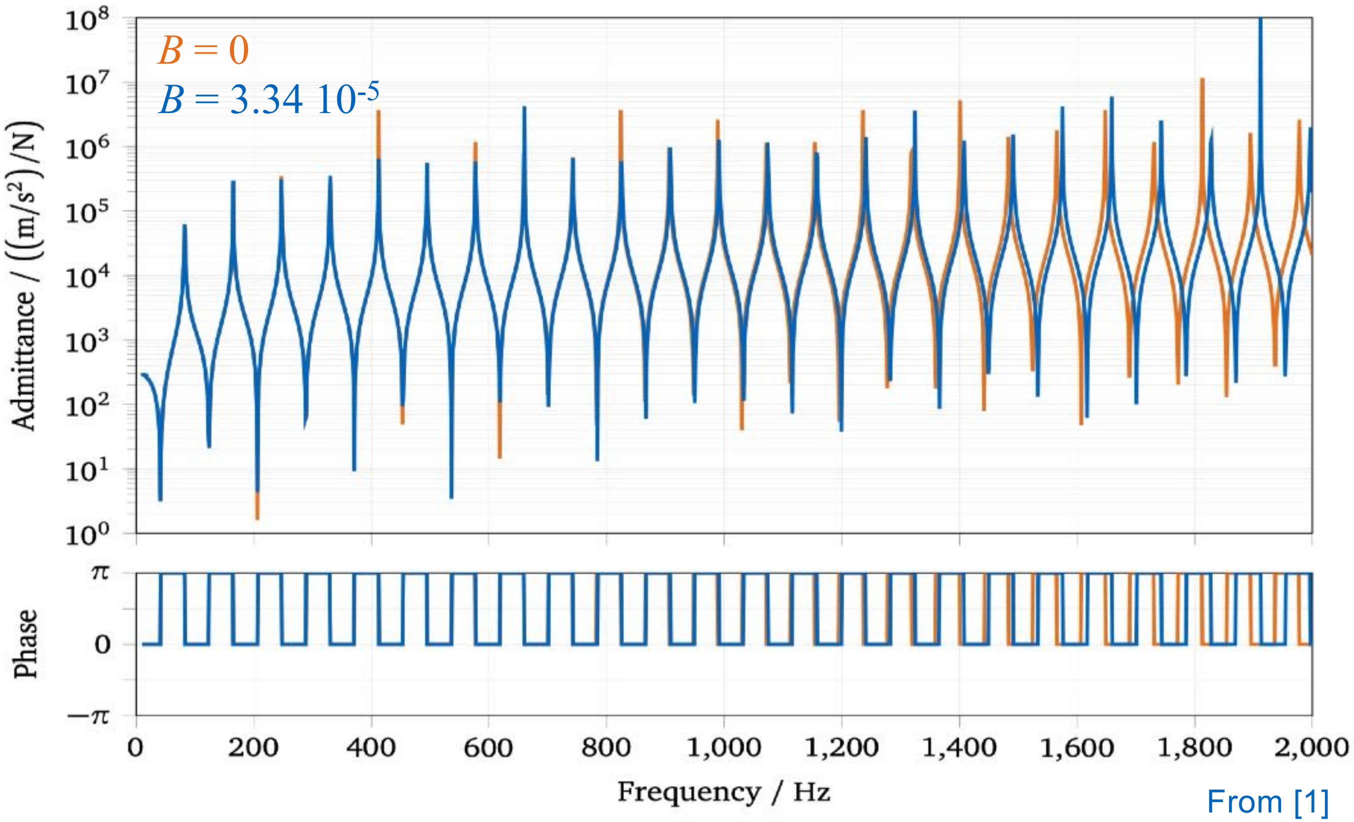
[1] Tim Schnabel "Dynamic Substructuring of a Guitar", Semester Thesis TUM School of Engineering and Design, October 2025

Analytical admittance of string (E - 82.41 Hz)



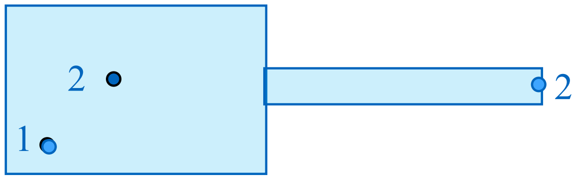


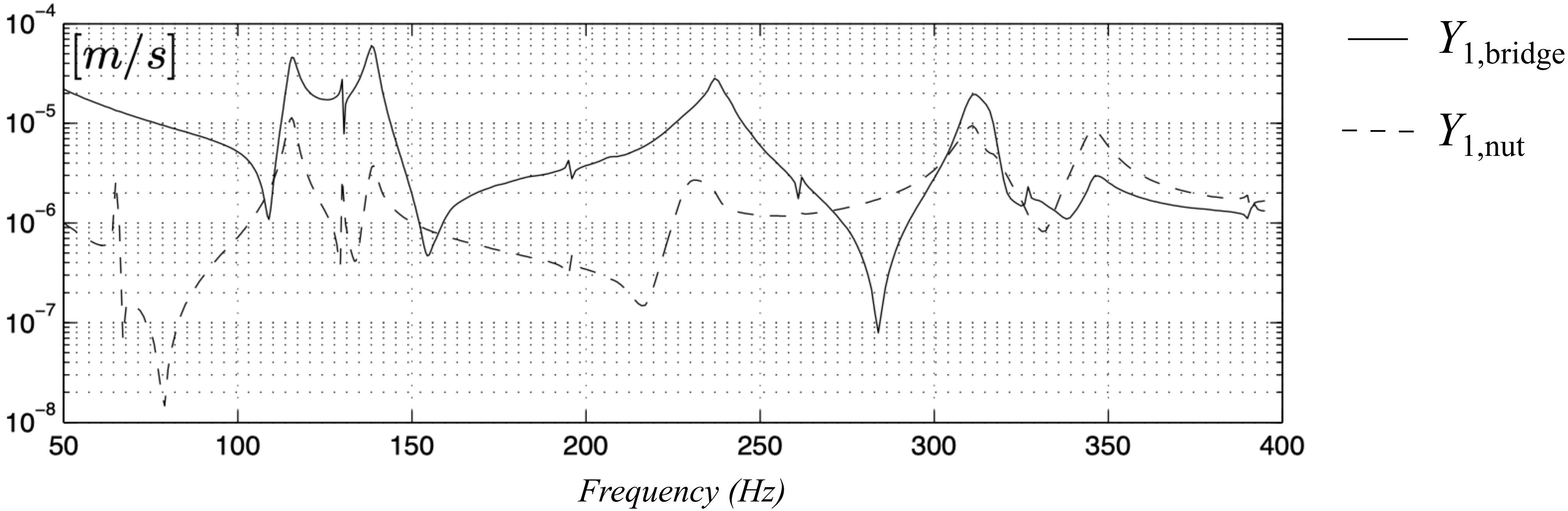
inharmonictiy factor $B = \dfrac{\pi^2 E I_y}{L^2 F_x}$



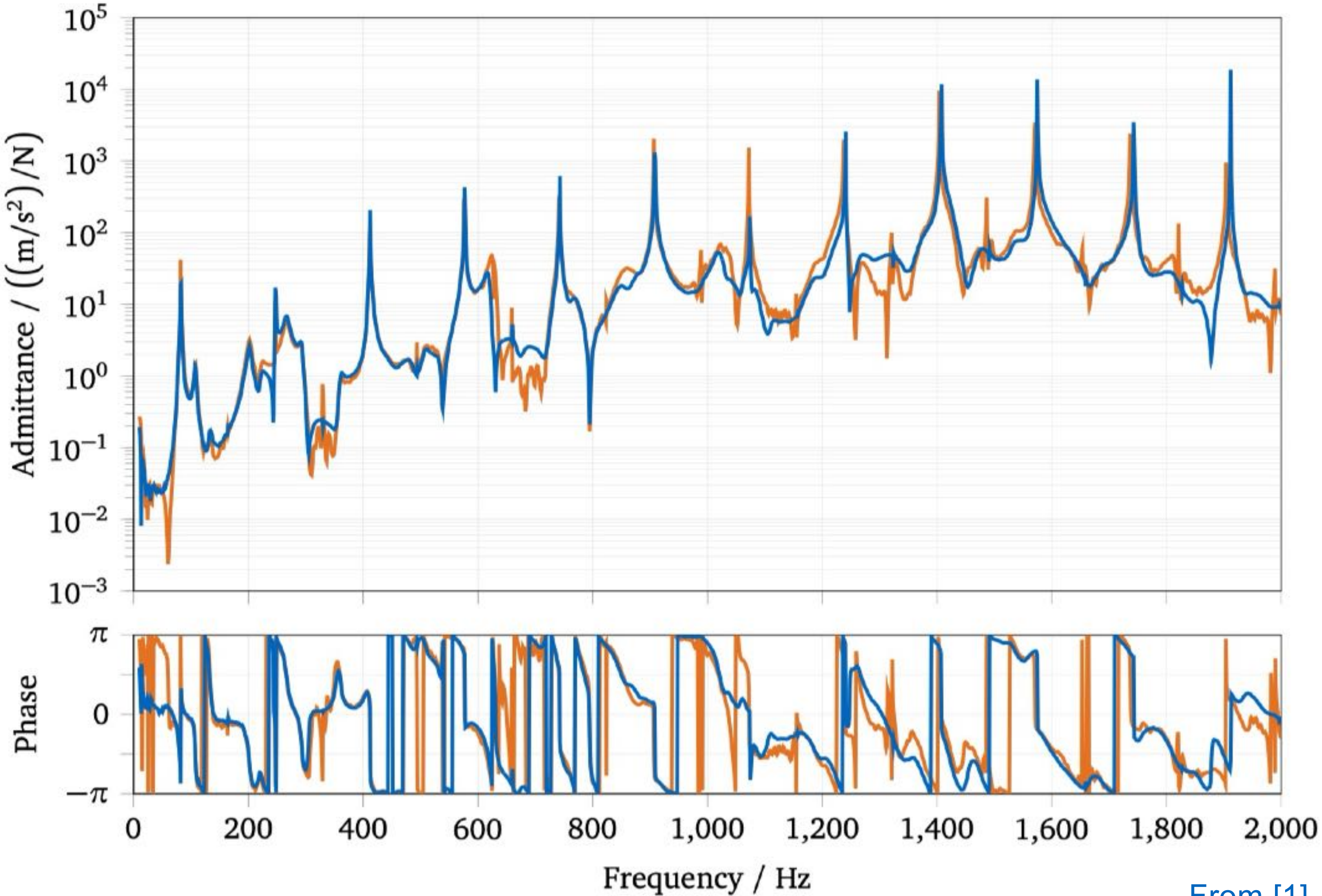
From [1]

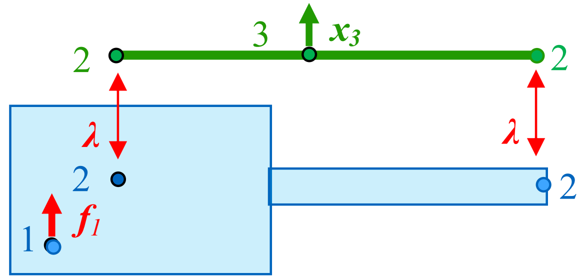
Measured admittance of the body alone





Validation (middle of the string measured by Laser Vibrometer)





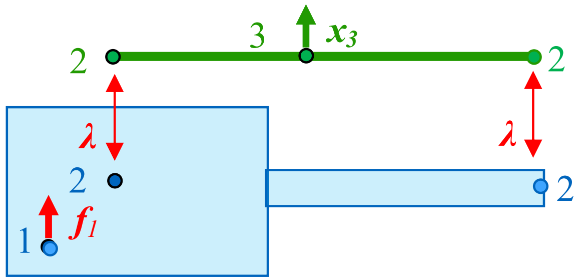
Validation (assembled system)

Substructuring (FBS)

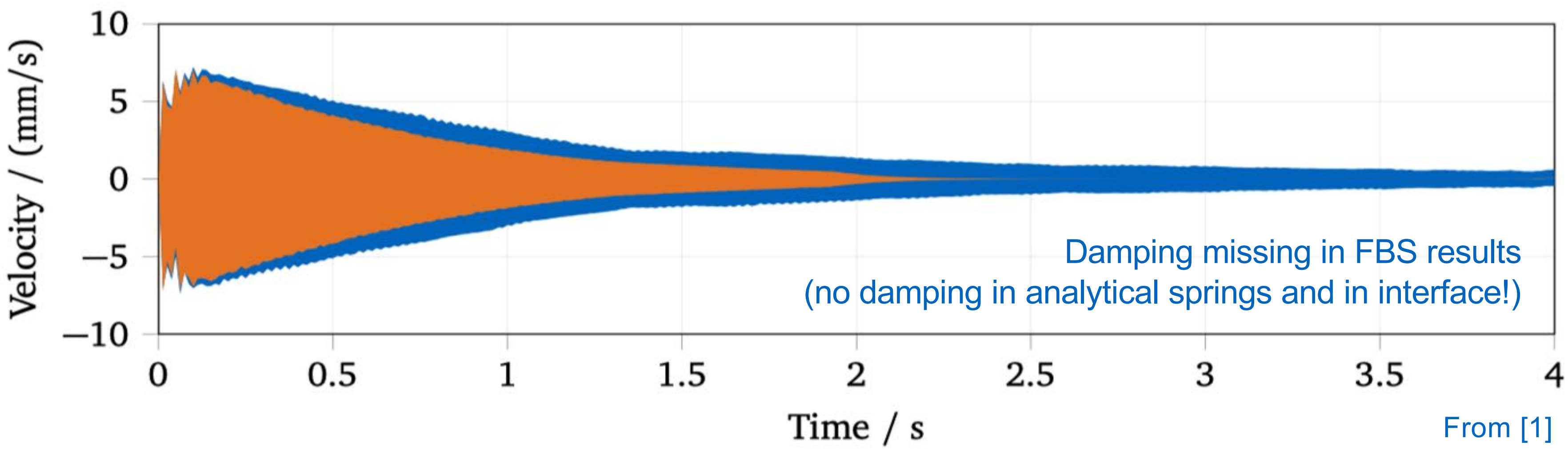
From [1]

Auralisation (impulse input, IFFT)

Validation (assembled system)



Substructuring (FBS)



From [1]

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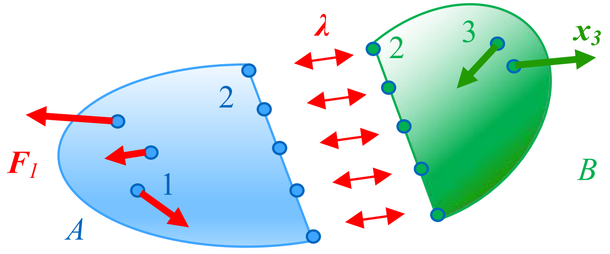
- Frequency-Based Substructuring (FBS)
    - Dual assembly of components (example of a guitar)
    - **Weakening of the interface compatibility: virtual point transformation (ex. AM structure)**
- Decoupling
    - Transmission simulator
    - Joint identification
- The concept of blocked forces
- Things not discussed, but certainly interesting …
- Current research directions

Weakening of the interface compatibility: the virtual point transformation



$$x_3^B = Y_{32}^B (Y_{22}^A + Y_{22}^B)^{-1} Y_{21}^A f_1^A$$

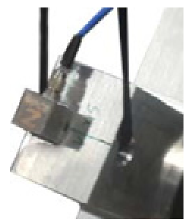
Internal interface forces λ to close the gap

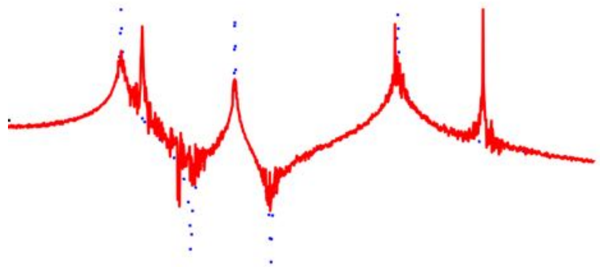
Frequency-Based Substructuring (FBS)

Computation of the interface forces requires summing interface admittances and inverting.
If Y_{22}^A and/or Y_{22}^B are obtained from measurements, they are always inexact due to

- calibration errors of the force and displacement sensors
- noise in the signal and data acquisition errors (windowing, filters ….)
- added mass of the vibration sensors, damping of sensor cables …
- errors in the positioning/orientation of the sensors

→ Errors amplified in the FBS formulas (spurious peaks, significant noise …)

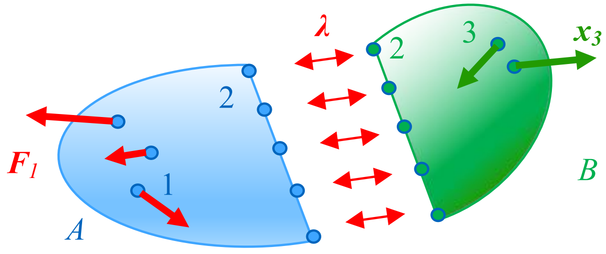




D. J. Rixen. How measurement inaccuracies induce spurious peaks in frequency based substructuring. In IMAC-XXVII: International Modal Analysis Conference, Orlando, FL, Bethel, CT, February 2008.

See also Paper 21

Weakening of the interface compatibility: the virtual point transformation



$$\boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}\boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$$

Internal interface forces $\boldsymbol{\lambda}$ *to close the gap*

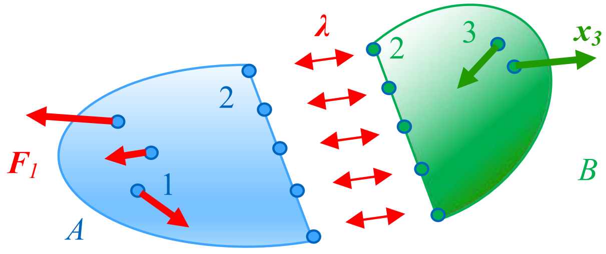
Frequency-Based Substructuring (FBS)

$(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}$ by truncated Singular Value Decomposition (SVD) or similar variant

+ The kept singular vectors can be interpreted as a reduced space for the interface forces (Lagrange multipliers)

– Rarely successful since does not restore physicality in the data and there is no clear indictor of where to truncated

Weakening of the interface compatibility: the virtual point transformation



$$\boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}\boldsymbol{Y}_{21}^A\boldsymbol{f}_1^A$$

Internal interface forces $\boldsymbol{\lambda}$ to close the gap

Frequency-Based Substructuring (FBS)

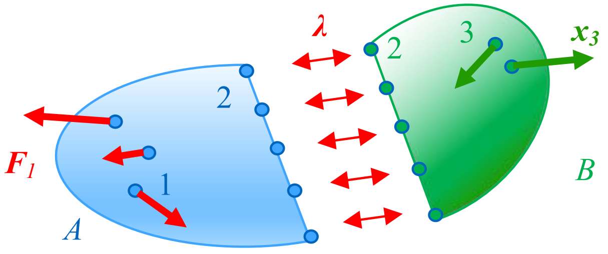
$(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}$ by modal identification and re-synthesis of $\boldsymbol{Y}_{22}^A$ and $\boldsymbol{Y}_{22}^B$

+ Introduces physicality in FRFs before coupling (helps alleviating spurious peaks)
+ Can save time since part of the FRF needed for FBS can be synthesized
+ Can be seen as a ***Modal-Based-Substructuring*** (MBS) for which good literature exist

– Tedious and hard to apply in case of high modal density or large damping
– Requires weakening (see later) of interface since non-compatible subspace on interface

M. S. Allen, D. Rixen, M. van der Seijs, P. Tiso, T. Abrahamsson, and R. L. Mayes. Substructuring in Engineering Dynamics, volume 594 of CISM International Centre for Mechanical Sciences. Springer, 2020.

Weakening of the interface compatibility: the virtual point transformation



$$\boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}\boldsymbol{Y}_{21}^A\boldsymbol{f}_1^A$$

Internal interface forces λ to close the gap

Frequency-Based Substructuring (FBS)

$(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}$ by filtering the FRFs with automatic procedures like PRANK

Principle Response Functions and Hankel realization

- \+ Restores some physicality in the data
- \+ Usually efficient to reduce noise
- \+ Some automatic tuning of the filter is possible

- – The filtering setting might still be application dependent

Trainotti, F., Klaassen, S. W. B., Bregar, T., & Rixen, D. J. (2025). "PRANK: A Singular Value Based Noise Filtering of Multiple Response Datasets for Experimental Dynamics." Experimental Techniques, 49. https://doi.org/10.1007/s40799-024-00774-1.

Contents

- Motivation and some basics on dynamic matrices and assembly
- Frequency-Based Substructuring (FBS)
  - Dual assembly of components (example of a guitar)
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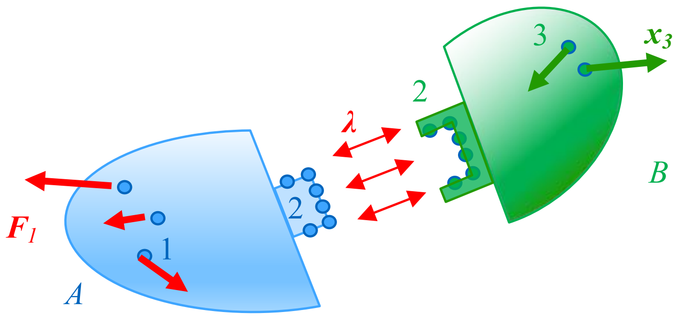
  - Transmission simulator
  - Joint identification
- The concept of blocked forces
- Things not discussed, but certainly interesting …
- Current research directions

Trying to “cure” the inverse or the FRFs in the inverse using modal identification or filtering does often not work well. It strongly helps to introduce physical a-priori knowledge of the interface behavior!

Assume that the interface motion is nearly a rigid body motion:

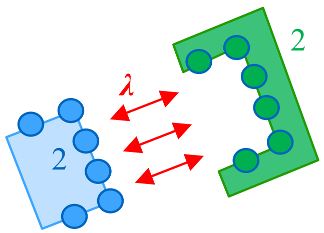


$$\boldsymbol{x}_3^B = \mathbf{Y}_{32}^B \left(\mathbf{Y}_{22}^A + \mathbf{Y}_{22}^B\right)^{-1} \mathbf{Y}_{21}^A \boldsymbol{f}_1^A$$

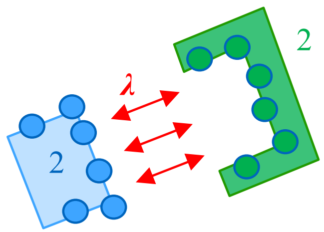
Internal interface forces $\boldsymbol{\lambda}$ *to close the gap*

Frequency-Based Substructuring (FBS)

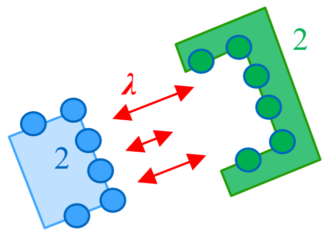
If one then tries to connect all dofs measured on the interface, then any small error in the measurements will generate very large and erroneous interface forces:



Correct interface forces when dofs move as a rigid body

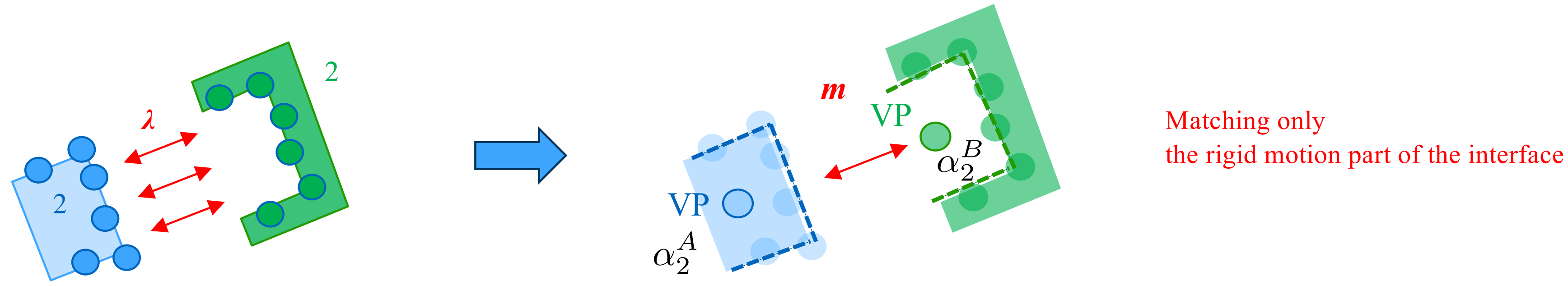


Dofs motion according to measured admittance when correct interface forces are applied



Interface forces needed to ensure compatibility according to measured admittances

To avoid big errors, estimate an average and impose that only the average motion should be compatible, leaving detailed dynamics (badly measured) unmatched, so they do not contribute to determining the interface forces:



The interface motion on each side is represented by a virtual point (VP) that has 6 dofs / 3 forces and 3 moments

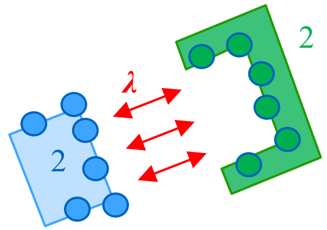
$$\boldsymbol{x}_2^A = \boldsymbol{R}_2^A \alpha_2^A + \mu^A$$

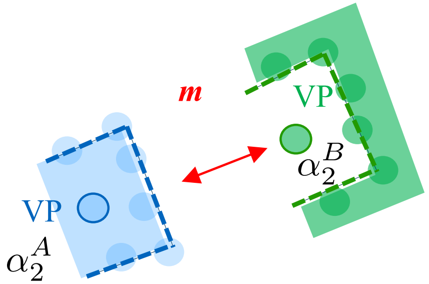
Asking the error μ^A to be orthogonal to the rigid body modes (so 0 in an average sense):

$$\boldsymbol{R}_2^{A^T} \mu^A = 0 \rightarrow \boldsymbol{R}_2^{A^T} \boldsymbol{x}_2^A = \boldsymbol{R}_2^{A^T} \boldsymbol{R}_2^A \alpha_2^A$$

$$\rightarrow \alpha_2^A = \left(\boldsymbol{R}_2^{A^T} \boldsymbol{R}_2^A\right)^{-1} \boldsymbol{R}_2^{A^T} \boldsymbol{x}_2^A = \boldsymbol{T}_{VPT}^A \boldsymbol{x}_2^A$$

Transformation matrix that computes
in a least-square sense the rigid motion amplitudes





Reconstructs physical forces with minimum norm

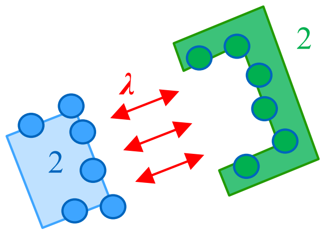
$$\lambda^A = \boldsymbol{R}^A \left(\boldsymbol{R}^{A^T} \boldsymbol{R}^A\right)^{-1} \boldsymbol{m}$$

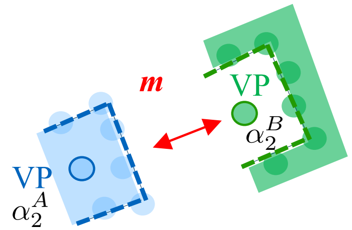
This virtual point transformation (VPT) also be applied on the B side.

$$\begin{bmatrix} \boldsymbol{Z}^A & 0 & \begin{bmatrix} 0 \\ \boldsymbol{I} \end{bmatrix} \\ 0 & \boldsymbol{Z}^B & \begin{bmatrix} -\boldsymbol{I} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \boldsymbol{I} \end{bmatrix} & \begin{bmatrix} -\boldsymbol{I} & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1^A \\ \boldsymbol{u}_2^A \\ \boldsymbol{u}_2^B \\ \boldsymbol{u}_3^B \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_1^A \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \boldsymbol{Z}^A & 0 & \begin{bmatrix} 0 \\ \boldsymbol{T}_{VPT}^{A^T} \end{bmatrix} \\ 0 & \boldsymbol{Z}^B & \begin{bmatrix} -\boldsymbol{T}_{VPT}^{B^T} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \boldsymbol{T}_{VPT}^A \end{bmatrix} & \begin{bmatrix} -\boldsymbol{T}_{VPT}^B & 0 \end{bmatrix} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1^A \\ \boldsymbol{u}_2^A \\ \boldsymbol{u}_2^B \\ \boldsymbol{u}_3^B \\ \boldsymbol{m} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_1^A \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and the dual interface problem becomes

$$(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)\boldsymbol{\lambda} = -\boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A \Rightarrow \left(\boldsymbol{T}_{VPT}^A \boldsymbol{Y}_{22}^A \boldsymbol{T}_{VPT}^{A^T} + \boldsymbol{T}_{VPT}^B \boldsymbol{Y}_{22}^B \boldsymbol{T}_{VPT}^{B^T}\right) \boldsymbol{m} = \boldsymbol{T}_{VPT}^A \boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$$





Enforcing then the compatibility only for the VPs and using the resulting forces $\boldsymbol{m}$ as interface forces, one obtains to the modified FBS formula

$$\boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}\boldsymbol{Y}_{21}^A\boldsymbol{f}_1^A$$

$$\boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B\boldsymbol{T}_{VPT}^{B^T}\left(\boldsymbol{T}_{VPT}^A\boldsymbol{Y}_{22}^A\boldsymbol{T}_{VPT}^{A^T} + \boldsymbol{T}_{VPT}^B\boldsymbol{Y}_{22}^B\boldsymbol{T}_{VPT}^{B^T}\right)^{-1}\boldsymbol{T}_{VPT}^A\boldsymbol{Y}_{21}^A\boldsymbol{f}_1^A$$

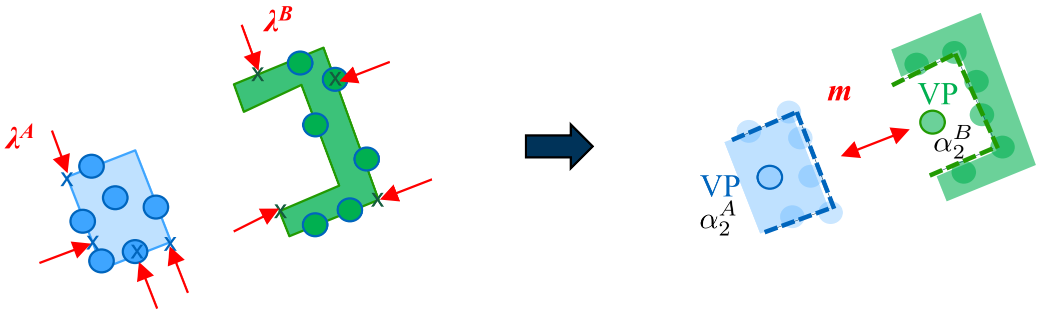
Internal interface $\boldsymbol{m}$ *to make the interface compatible in average*
→ Much better conditioned !

weakening of the interface compatibility that avoids small measurement errors to affect too much the assembly result (technique similar to RBE3 elements in commercial codes).

$$\boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B \boldsymbol{T}_{VPT}^{B^T} \left(\boldsymbol{T}_{VPT}^A \boldsymbol{Y}_{22}^A \boldsymbol{T}_{VPT}^{A^T} + \boldsymbol{T}_{VPT}^B \boldsymbol{Y}_{22}^B \boldsymbol{T}_{VPT}^{B^T} \right)^{-1} \boldsymbol{T}_{VPT}^A \boldsymbol{Y}_{21}^A \boldsymbol{f}_1^A$$

Remarks:

1. No need anymore to measure on matching locations! Simplifies significantly the measurement campaign !
2. Locations for the force input and for the displacements on interface can be different !

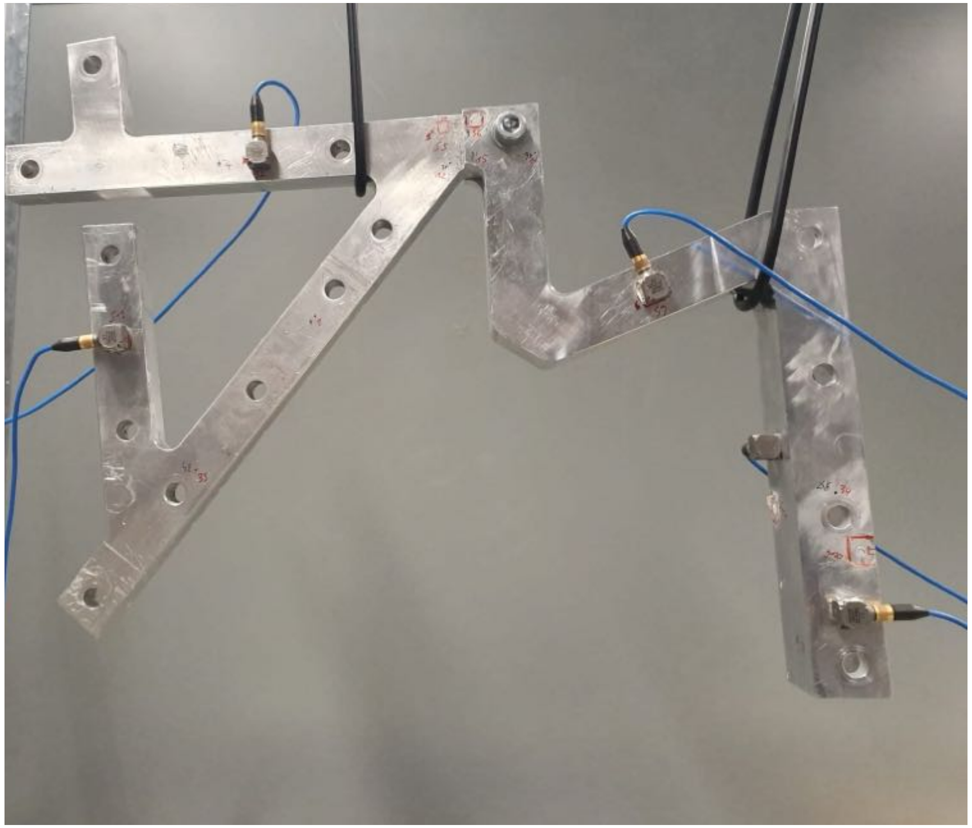


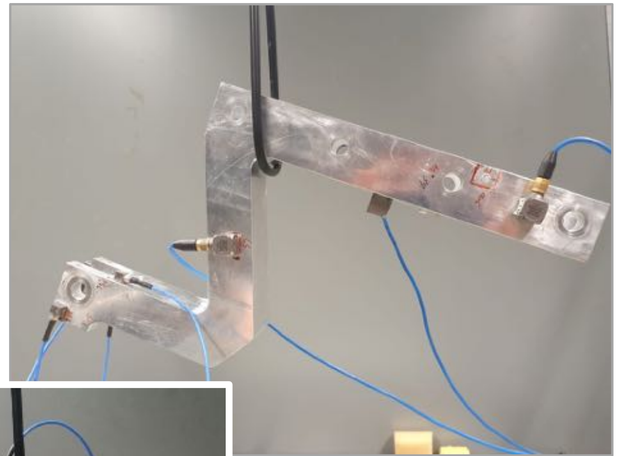
3. Can be generalized to any kind of interface deformation shapes (e.g obtained form FE or from measurements)
4. One can choose different representations for different frequency ranges
5. If one does not know what basis to choose, one can build a large basis and compute the filters using sparsity promoting inverses.

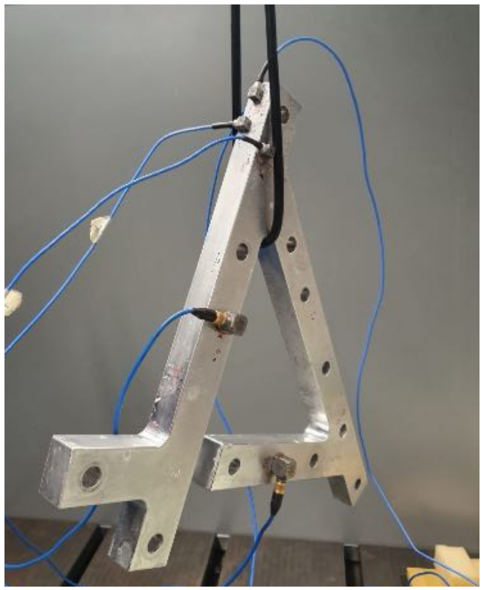
[1]M. S. Allen, D. Rixen, M. van der Seijs, P. Tiso, T. Abrahamsson, and R. L. Mayes. *Substructuring in Engineering Dynamics*, volume 594 of *CISM International Centre for Mechanical Sciences*. Springer, 2020.

[2]Trainotti, F., Korbar, J., Zobel, O., Kreutz, M., Ocepek, D., Cepon, G., & Rixen, D. J. (2025). "On the use of sparse regression strategies in frequency-based substructuring." In Structural Engineering Dynamics – Proceedings of ICEDyn 2025. Springer. Accepted for publication.

Example of an assembly by Frequency Based Substructuring (FBS)







***PyFBS**: open-source DS*

Numerical modelling

- FEM import
- Model reduction
- Eigenvalue extraction
- FRF synthesis

Experimental modelling

- Data Import (**.uff, *.hdf, …*)
- Sensor/impact positioning
- Interface reduction: VPT…
- ODS animation
- Mode shape animation

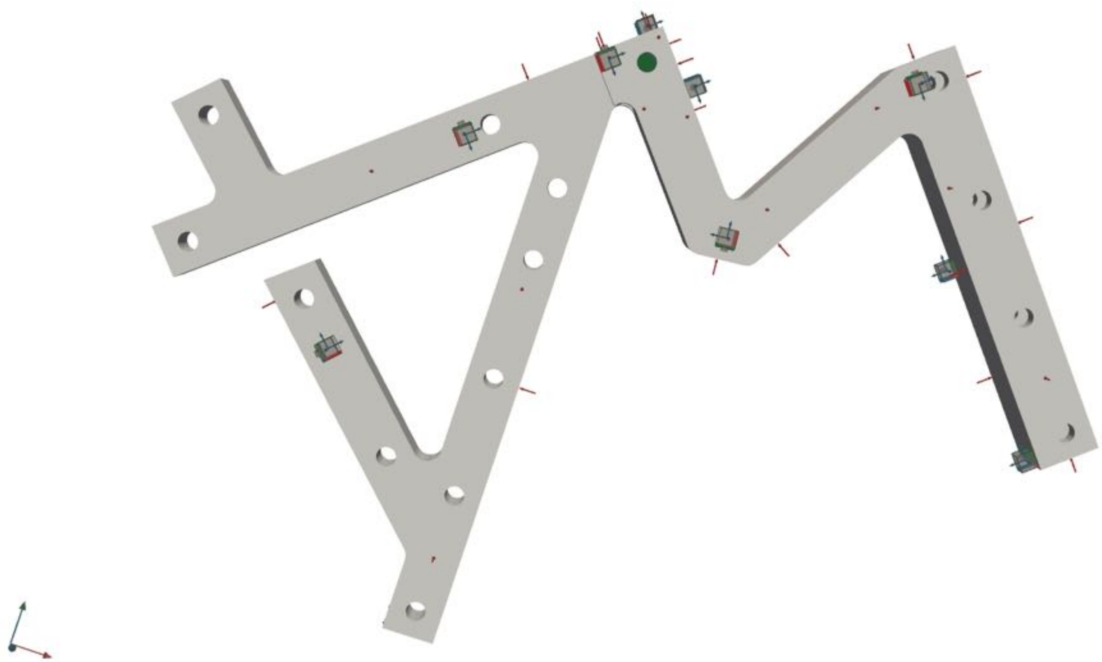
Hybrid modelling

- System equivalent model mixing (SEMM)
- System equivalent reduction and expansion process (SEREP)

https://gitlab.com/pyFBS/pyFBS

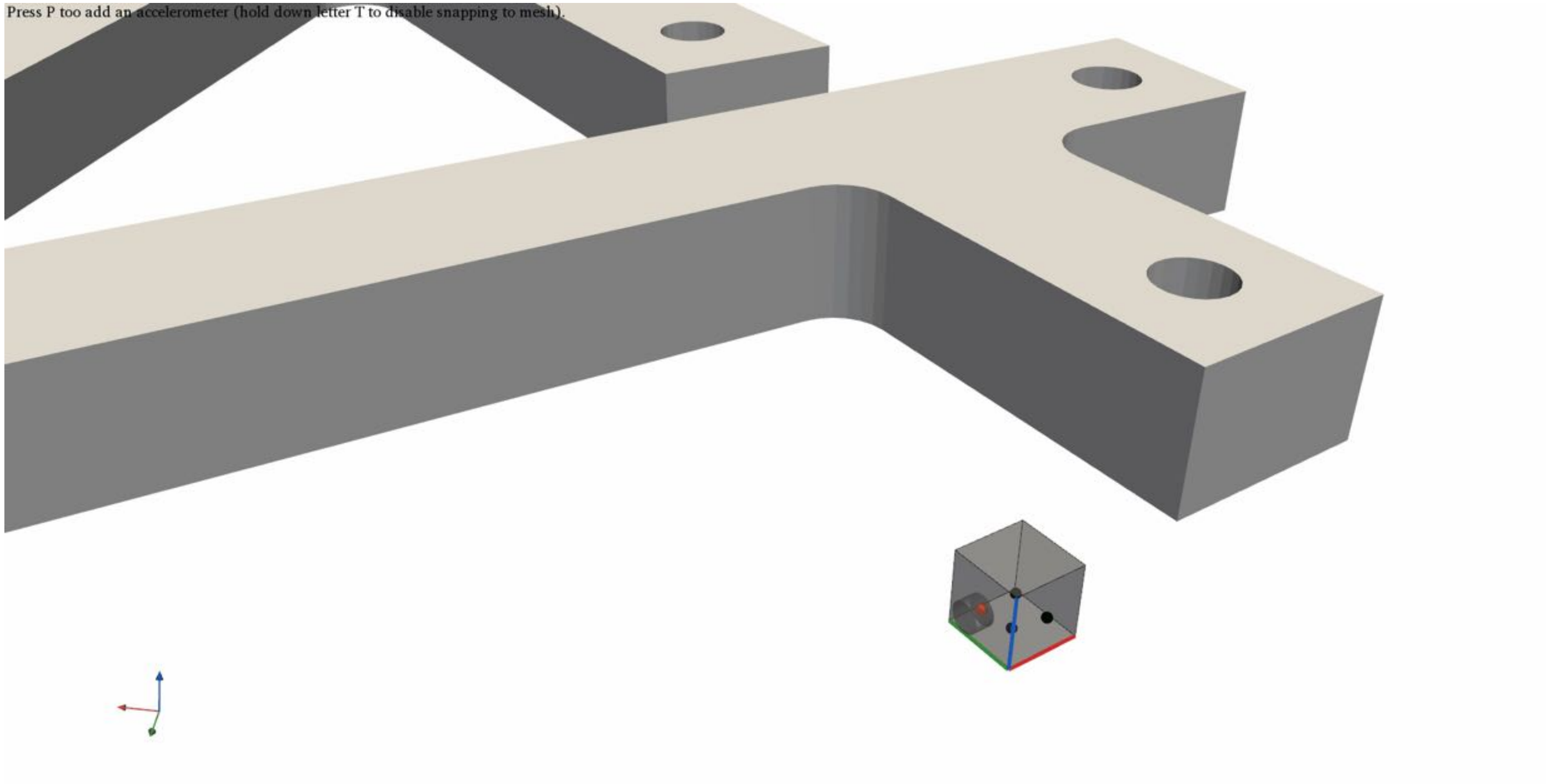
PyFBS: open-source DS

- Easy organization of data
- Design of Experiment using CAD models

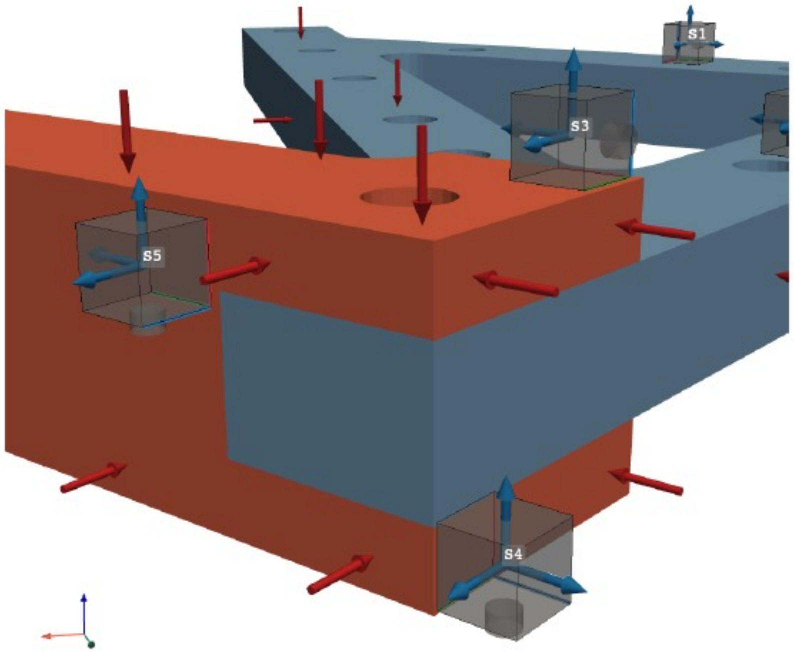


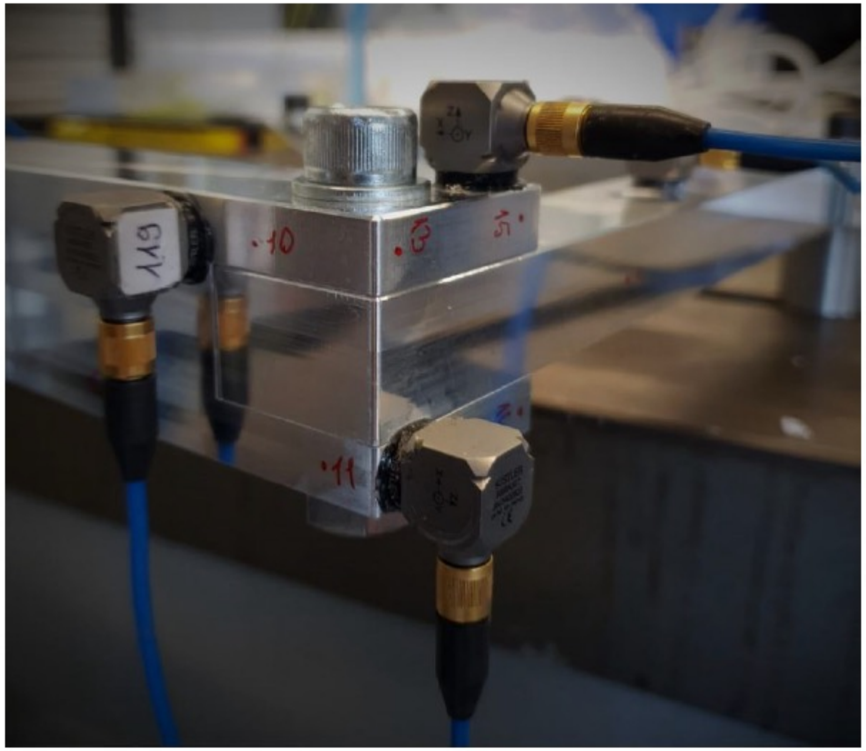
PyFBS: open-source DS

Interactive DOE

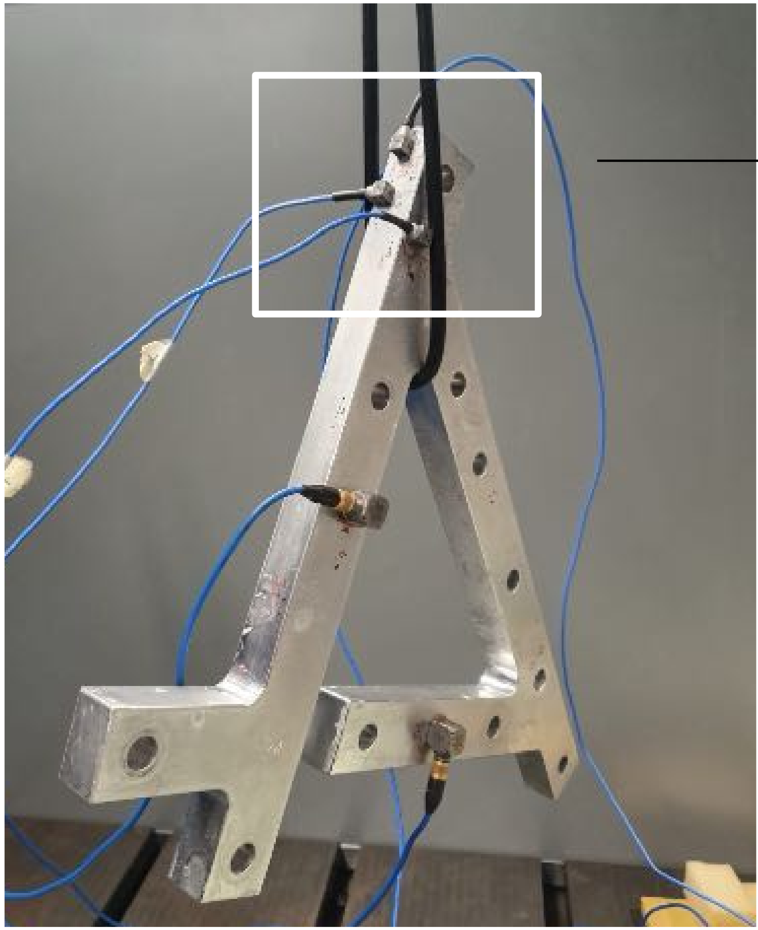


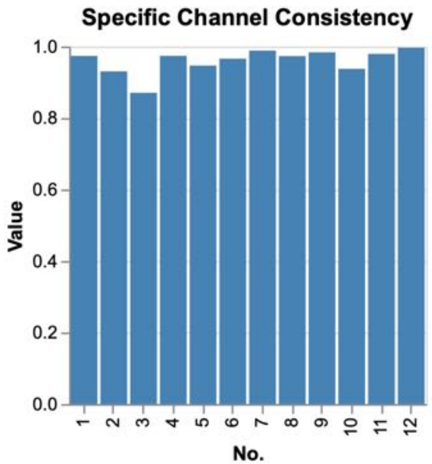
PyFBS: open-source DS

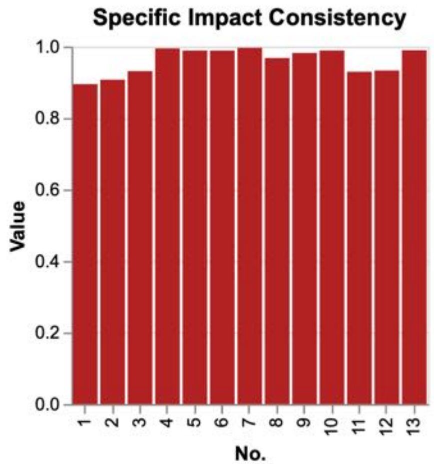




Verifying rigidity of interfaces

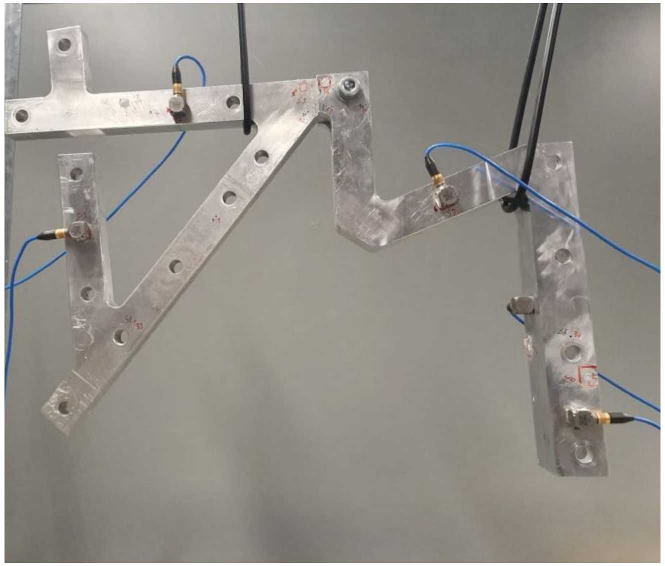


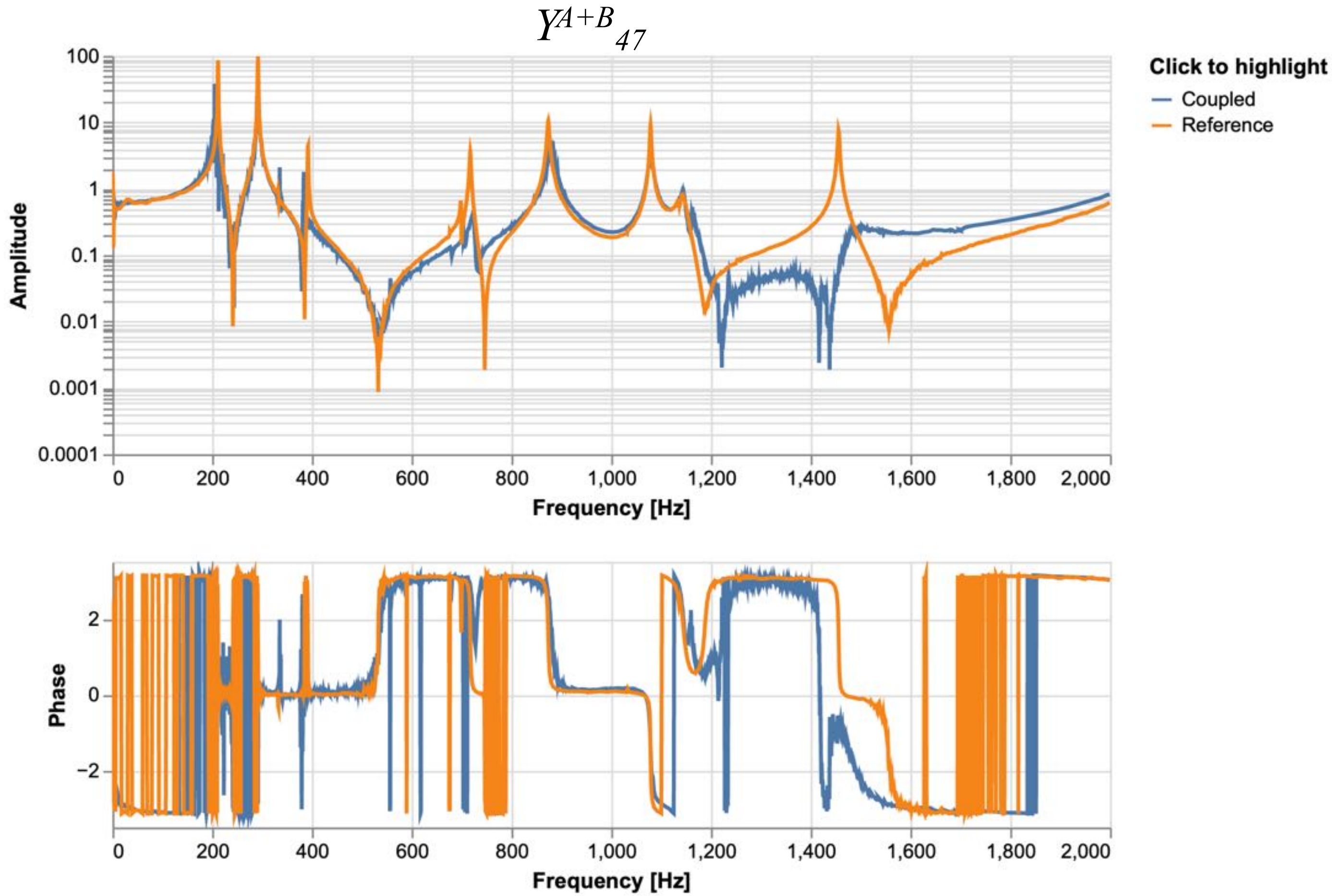




M. van der Seijs. *Experimental Dynamic Substructuring, Analysis and DesignStrategies for Vehicle Development*. PhD thesis, Delft University of Technology, Delft, The Netherlands, June 2016.

Assemble and compare to full structure





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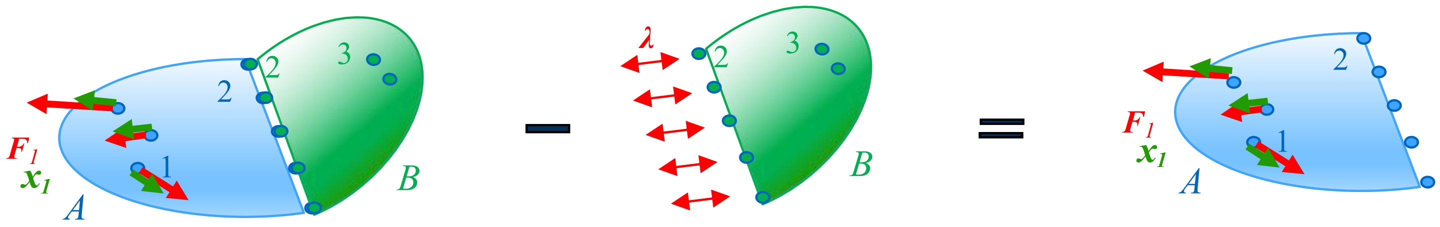
  - Transmission simulator
  - Joint identification
- The concept of blocked forces
- Things not discussed, but certainly interesting …
- Current research directions

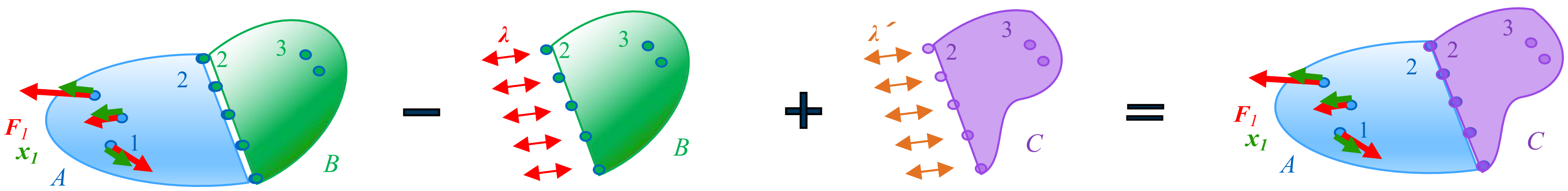
Decoupling

Interesting when:

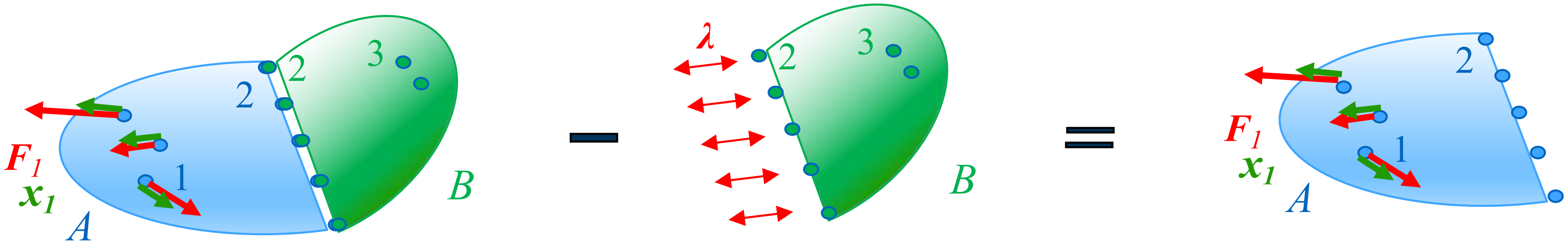
- a component A has too big amplitudes during the test or cannot be well excited because it is too small



- to analyse dynamic changes when a component is replaced by another:



Decoupling = adding a "negative" substructure B to the assembly $A+B$:



$$\boldsymbol{Y}_{11}^{A} \;,\; \boldsymbol{Y}_{21}^{A}$$

$$\boldsymbol{x}_1^A = \left(\boldsymbol{Y}_{11}^{A+B} - \boldsymbol{Y}_{12}^{A+B}\left(\boldsymbol{Y}_{22}^{A+B} - \boldsymbol{Y}_{22}^{B}\right)^{-1}\boldsymbol{Y}_{21}^{A+B}\right)\boldsymbol{f}_1^A$$

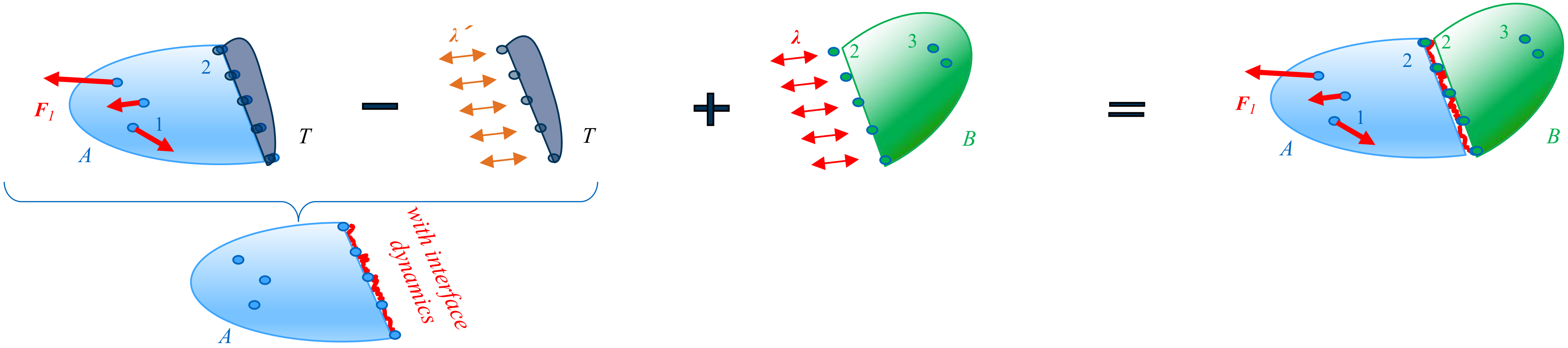
Frequency-Based
Decoupling

*Internal interface forces **λ** present in the assembly, that must be removed to have the behavior of A without coupling*

[1] *P. Sjövall and T. Abrahamsson. Substructure system identification from coupled system test data. Mechanical Sys. Sign Proc. , 22:1–260, 2007.*
[2] *W. D'Ambrogio, et al., The role of interface DoFs in decoupling of substructures based on the dual domain decomposition, Mech. Sys. Sign Proc. 24*
[3] S. Voormeeren et al. A family of substructure decoupling techniques based on a dual assembly approach. *Mech. Sys. Sign Proc.*, 27:379–396, 2012.

Decoupling: Transmission simulator

When the interface dynamics is important (bolted joints, glued interface …)

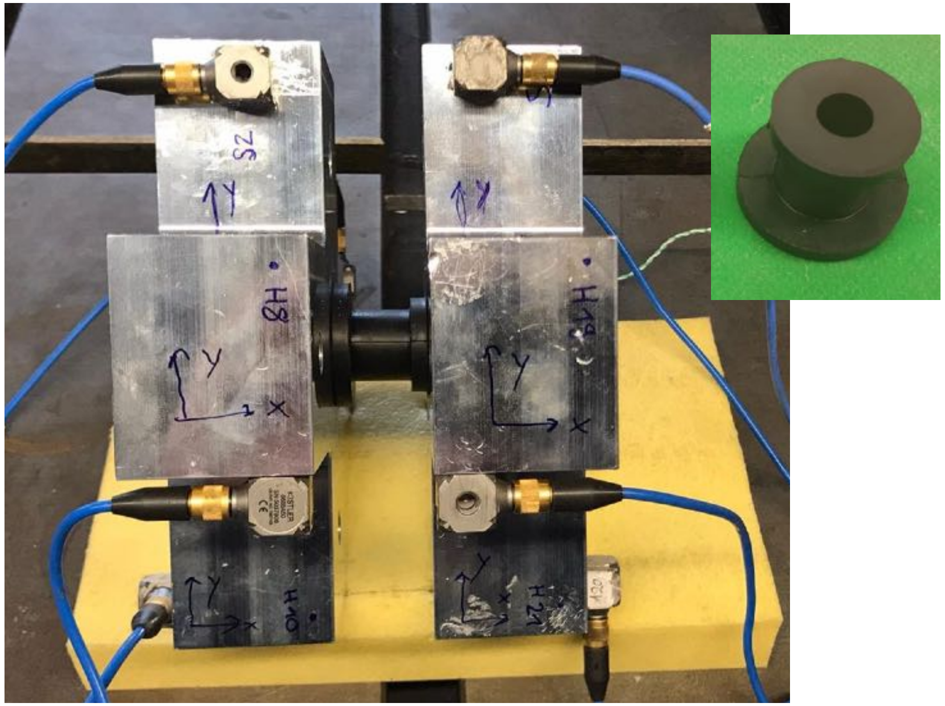


The transmission simulator should have an interface dynamics similar to what will be observed when substructure B is coupled

* The name “transmission simulator” was first coined in
R. L. Mayes and M. Arviso. Design studies for the transmission simulator method of experimental dynamic substructuring. In International seminar on modal analysis, ISMA, Leuven, 2010. KUL.

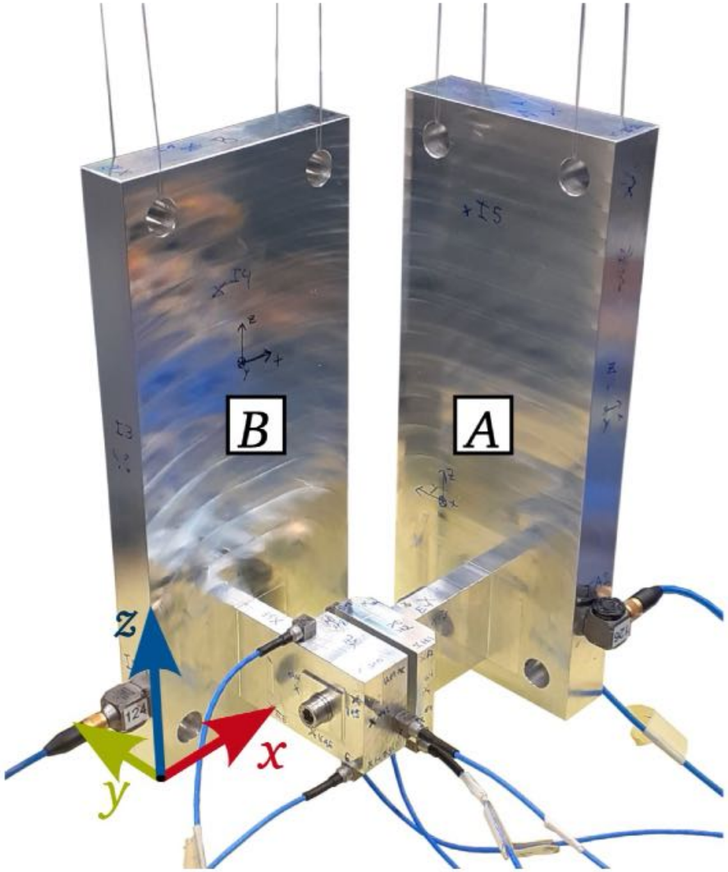
Decoupling: Joint identification

[2]

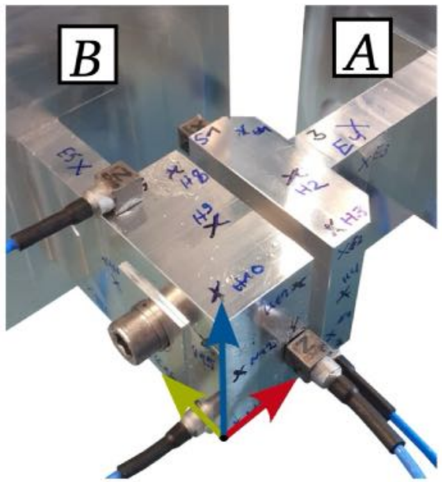


Brake, Matthew RW, and Pascal Reuß. "The Brake-Reuß beams: a system designed for the measurements and modeling of variability and repeatability of jointed structures with frictional interfaces." *The Mechanics of Jointed Structures: Recent Research and Open Challenges for Developing Predictive Models for Structural Dynamics* (2018): 99-107.

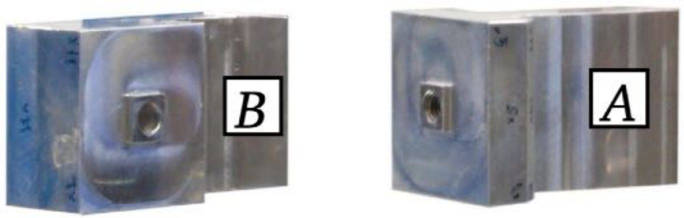
M. Haeussler, S. Klaassen, and D. Rixen. Experimental twelve degree of freedom rubber isolator models for use in substructuring assemblies. *Journal of Sound and Vibration*, 474:115253, 2020.



(a) Measurement setup with suspension.

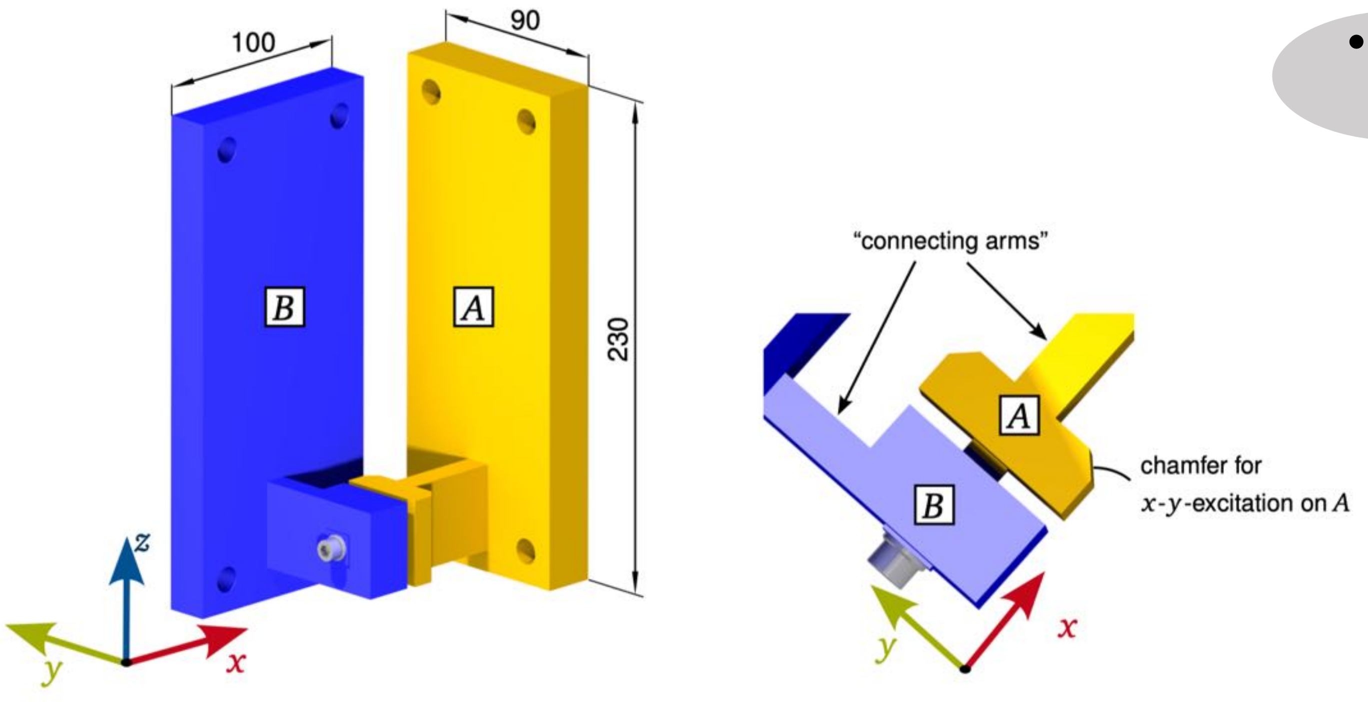


(b) Close-up of the assembled interface.

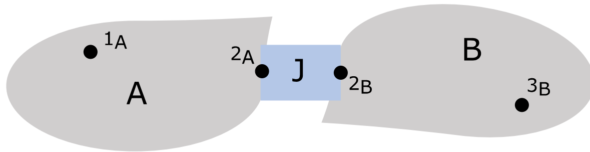


(c) Close-up on disassembled A and B.

Example: bolted joint



(a) Whole system. **(b)** Close-up of the contact.

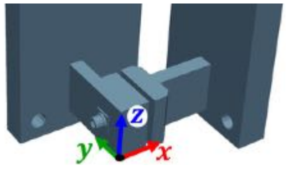


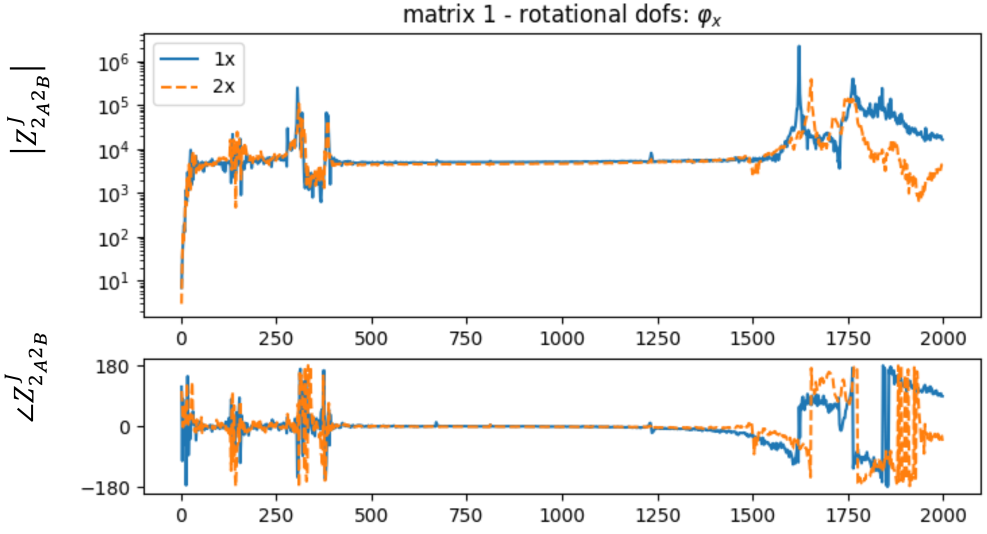
https://gitlab.com/pyFBS/pyFBS

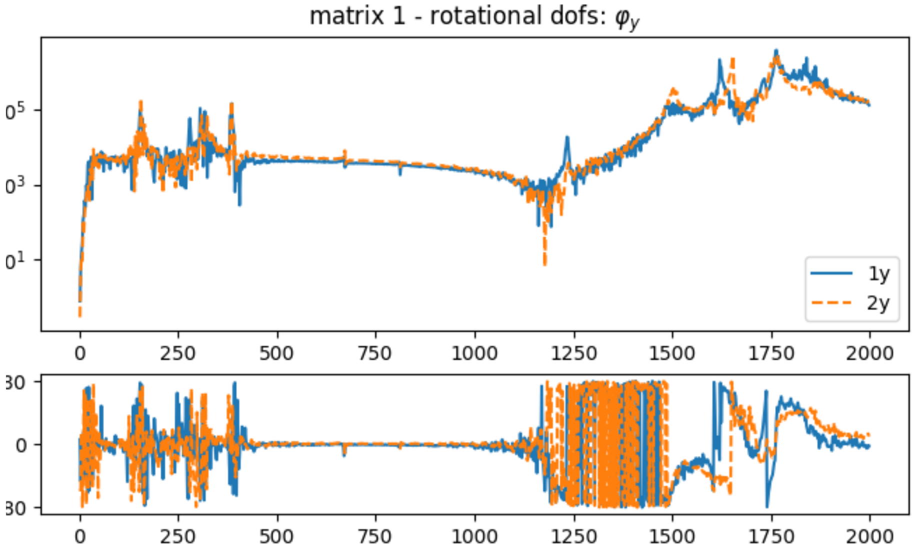
Different ways to perform the decoupling depending on available measurements and assumptions on joints

M. Kreutz, F. Trainotti, V. Gimpl, and D. J. Rixen. On the robust experimental multi-degree-of-freedom identification of bolted joints using frequency-based substructuring. *Mechanical Systems and Signal Processing*, 203:110626, 2023.

- Interface impedance (including repeatability): translational dofs







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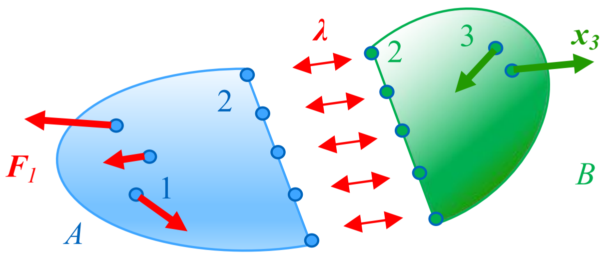
- Current research directions

The concept of blocked forces

How to measure the excitation source? Usually not possible (internal forces) ….



$$\boldsymbol{x}_3^B = \boldsymbol{Y}_{32}^B(\boldsymbol{Y}_{22}^A + \boldsymbol{Y}_{22}^B)^{-1}\boldsymbol{Y}_{21}^A\boldsymbol{f}_1^A$$

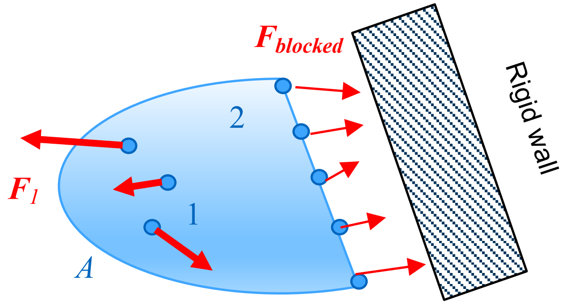
One idea would be to measure the interface force λ to characterize the source. Not a good idea, since interface force depend on the structure dynamics ! Change if substructure B changes.

A much better and independent characterisation of the source excitation can be done with "*blocked forces*".

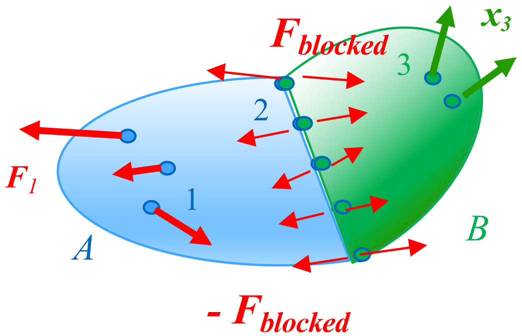
[1] M. V. van der Seijs, D. de Klerk, and D. J. Rixen. General framework for transfer path analysis: History, theory and classification of techniques. *Mechanical Systems and Signal Processing*, 68–69:217–244, 2016.
[2] D. J. Rixen, A. Boogaard, M. V. van der Seijs, G. van Schothorst, and T. van der Poel. Vibration source description in substructuring: A theoretical depiction. Mechanical Systems and Signal Processing
[3] M. Häußler. Modular sound & vibration engineering by substructuring - Listening to machines during virtual design. PhD thesis, Technical University Munich - Department of Mechanical Engineering, April 2021.

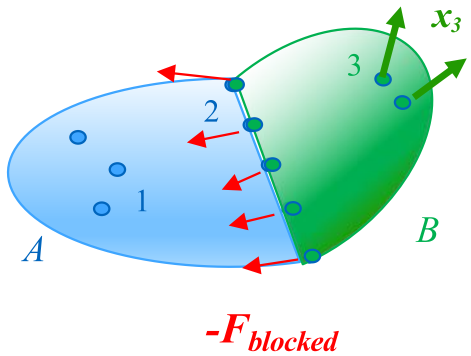
Imagine that one measures the force that the source substructure A would apply on a fully rigid wall

→ blocked forces $F_{blocked}$: forces that must be applied to fix the interface



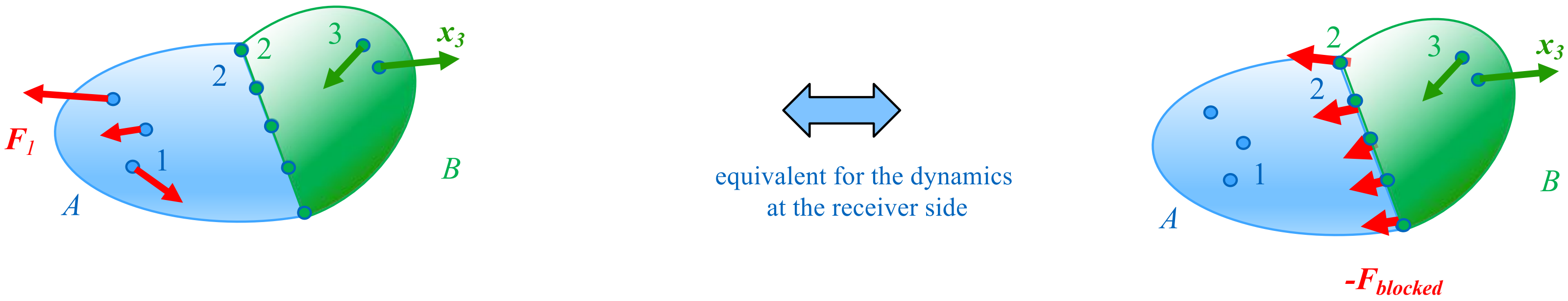
Let us now make the following "thought experiment":

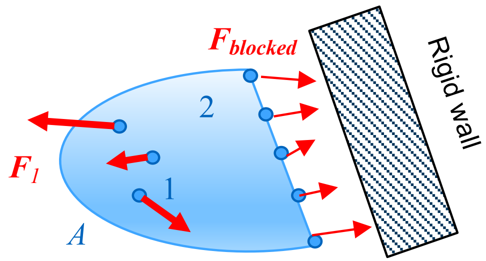




Apply $F_{blocked}$ and $-F_{blocked}$ on the interface.
This leaves obviously the dynamics unchanged

Remove $F_{blocked}$ and *the true source force* F_1
This also leaves the dynamics unchanged for the receiver B since the combination of these two forces keeps the interface still by definition of the blocked force.





Measuring blocked forces on a rigid rig is very hard:

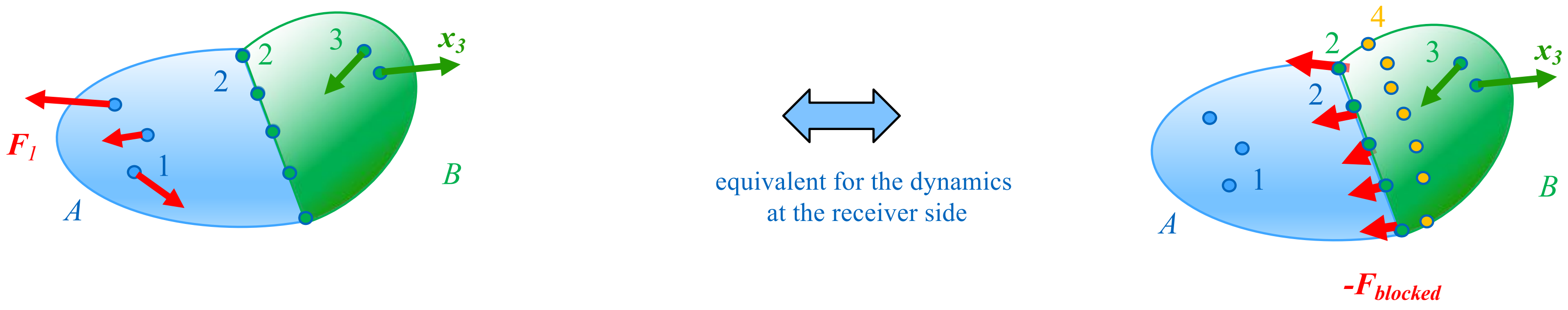
- A test rig in never fully rigid
- One needs to install several force sensor between the source and the test rig, thereby influencing the dynamics of the source …

Hence, identify *during the operation of the assembled system ("in-situ")* the force on the interface that would create the observed output in the receiver (so the blocked force):

A. Moorhouse, A. Elliott, and T. Evans. In situ measurement of the blocked force of structure-borne sound sources. *Journal of Sound and Vibration*, 325(4–5):679 – 685, 2009.

In-situ measurement of blocked forces

To identify the blocked force, we then place enough sensors in B close to the interface (called 4).



The equivalent force is then obtained as

$$\boldsymbol{F}_2^{eq} = -\boldsymbol{F}_{blocked} = (\boldsymbol{Y}_{42}^{A+B})^+ \boldsymbol{x}_4^{op}$$

FRF from interface forces to indicators

Vibration measured in operation at the indicators

Advantages:

- No need to disassemble A+B
- No need for a rigid test rig

Disadvantage:

- Needs measuring enough indicator signals
- Needs measuring $\boldsymbol{Y}_{42}^{A+B}$

Can be extended to vibroacoutics: M. Häußler. *Modular sound & vibration engineering by substructuring - Listening to machines during virtual design*. PhD thesis, Technical University Munich, April 2021.

Contents

- Motivation and some basics on dynamic matrices and assembly
- Frequency-Based Substructuring (FBS)
  - Dual assembly of components (example of a guitar)
  - Weakening of the interface compatibility: virtual point transformation (example of AM structure)
- Decoupling
  - Transmission simulator
  - Joint identification
- The concept of blocked forces
- **Things not discussed, but certainly interesting …**
- **Current research directions**

Not discussed here, but certainly interesting …

- Uncertainty propagation

[1] S. Voormeeren, D. de Klerk, and D. Rixen. Uncertainty quantification in experimental frequency based substructuring. *Mechanical Systems and Signal Processing*, 24(1):106 – 118, 2010.

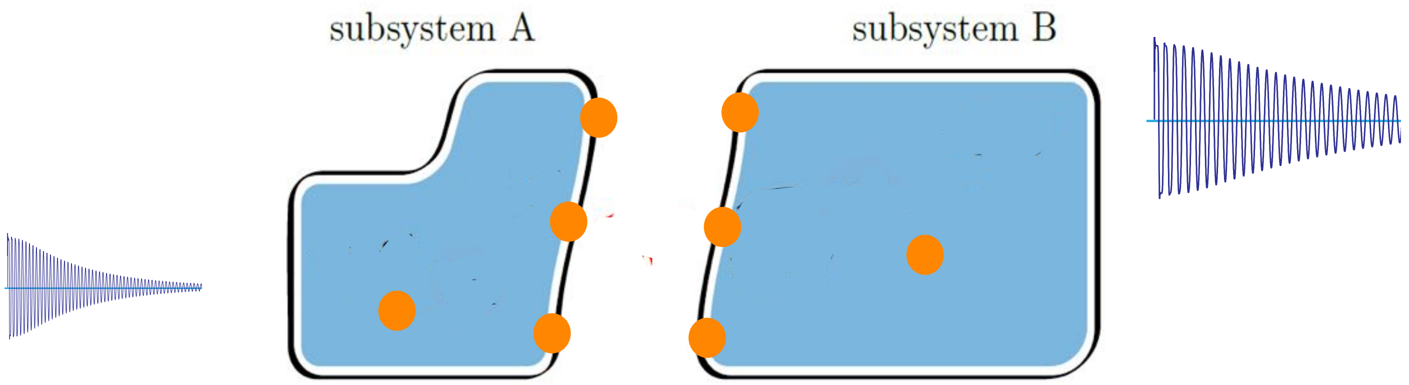
[2] Kammermeier, J. Mayet, and D. J. Rixen. Aleatoric uncertainty quantification: A state-space-based parametrization for hybrid substructuring. In IMAC-XXXVIII, January 2020. Society for Experimental Mechanics.

[3] J. Meggitt, A. Moorhouse, K. Wienen, and M. Sturm. A framework for the propagation of uncertainty in transfer path analysis. Mechanical Systems and Signal Processing, 2020.

[4] F. Trainotti, M. Haeussler, and D. Rixen. A practical handling of measurement uncertainties in frequency based substructuring. Mechanical Systems and Signal Processing, 144:106846, 2020.

Not discussed here, but certainly interesting …

- Experimental substructuring in the time domain (Impulse Based Substructuring - IBS)



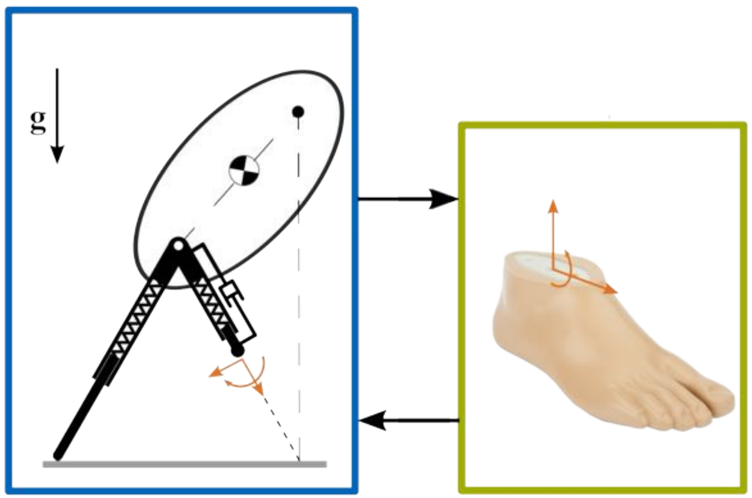
[1] D. J. Rixen. Substructuring technique based on measured and computed impulse response functions of components. In P. S. et al., editor, *ISMA*. K.U. Leuven, September 2010.

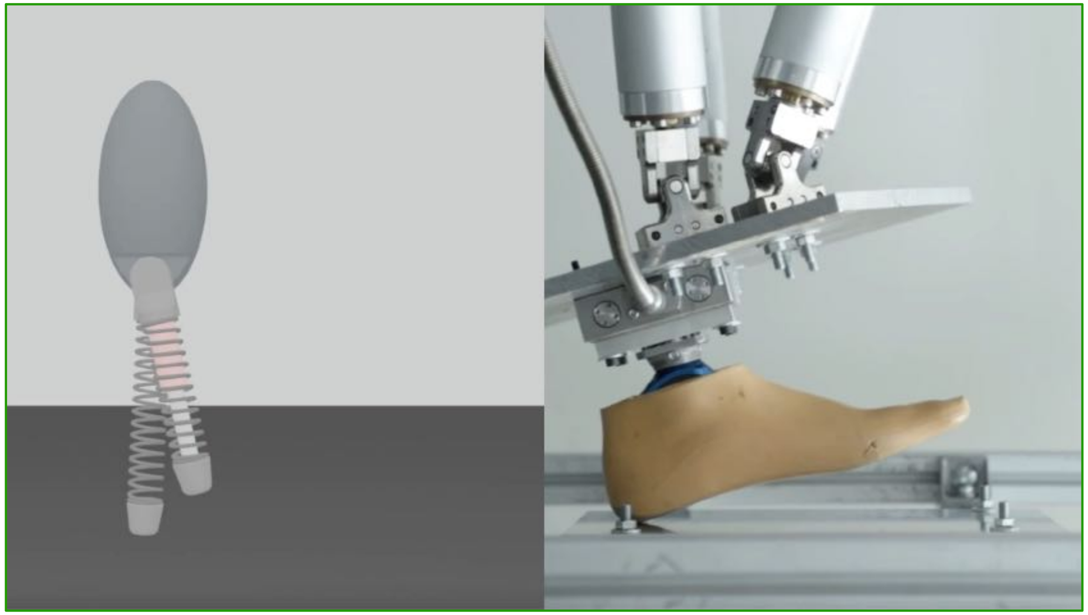
[2] O. M. Zobel, F. Trainotti, and D. J. Rixen. Toward 3d experimental impulse-based substructuring using the virtual point transformation. In IMAC-XLIII , February 2025. Society for Experimental Mechanics.

Not discussed here, but certainly interesting …

- Real-Time Hybrid Substructuring (RTHS)

0.25x





[1] C. Insam, L. D. H. Peiris, and D. J. Rixen. Normalized passivity control for hardware-in-the-loop with contact. *International Journal of Dynamics and Control*, 2021.
[2] C. Insam, L.-M. Ballat, F. Lorenz, and D. J. Rixen. Hardware-in-the-loop test of a prosthetic foot. Applied Sciences, 11(20), 2021.

Current Research directions

- Better understand the mathematics of weakening
- Improving experimental data for enhanced FBS and IBS
- Non-linear interfaces and components (see interesting work of Prof. Nevzat Özgüven)
- Exploiting Blocked forces for monitoring rotating systems.
- Experimental Substructuring with a human as a component
- ……

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